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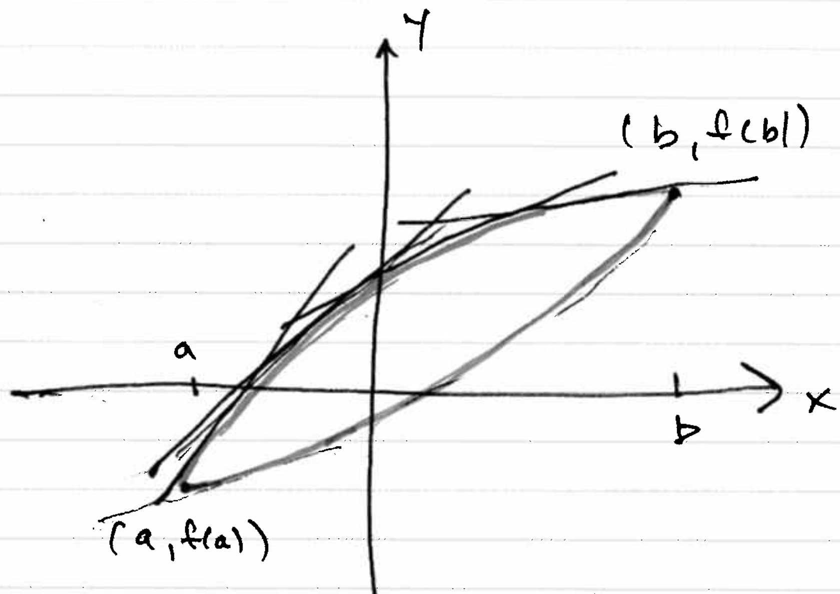
Memorial University of Newfoundland

§4.7 INTERVALS OF CONCAVITY & POINTS OF INFLECTION
4.8 CURVE SKETCHING (WITHOUT A TI-8x)

GIVEN:
(1) GRAPH JOINS $(a, f(a))$ TO $(b, f(b))$ CONTINUOUSLY
(2) $f(x)$ IS INCREASING ON (a, b)



THE "RED" ARC IS CONCAVE DOWN ON (a, b)

THE GREEN ARC IS CONCAVE UP ON (a, b).

DEFINITION

$f(x)$ IS CONCAVE UP ON THE OPEN INTERVAL I IF $f'(x)$ IS INCREASING "THROUGH" I.

$f(x)$ IS CONCAVE DOWN ON THE INTERVAL J IF $f'(x)$ IS DECREASING ON J.

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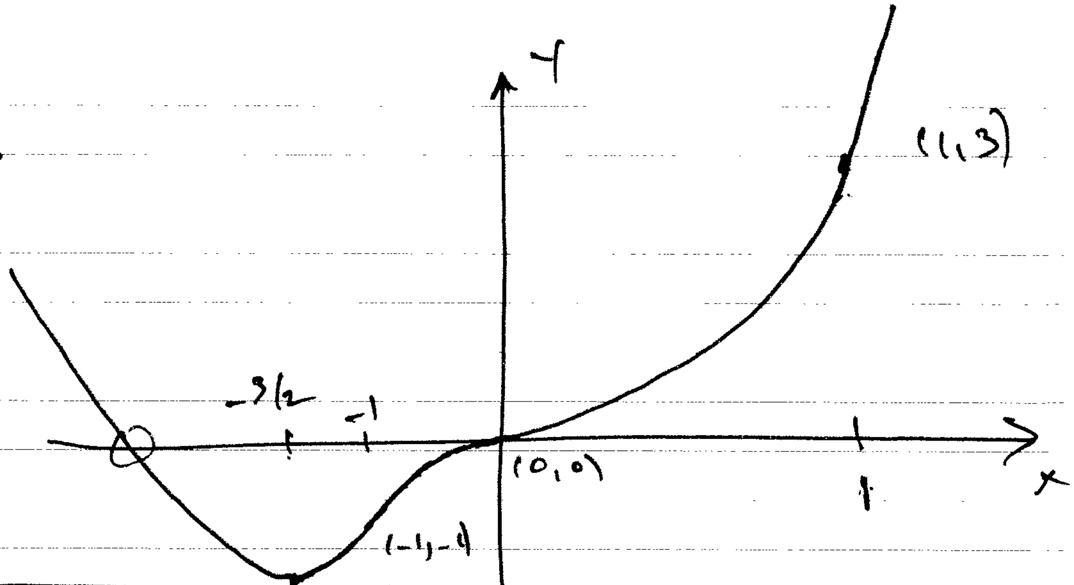
EXAMPLE

$f(x) = x^4 + 2x^3$



CONCAVE UP: $(-\infty, -1)$, $(0, \infty)$

CONCAVE DOWN: $(-1, 0)$

INTERVALS

THE POINTS $(-1,-1)$, $(0,0)$ ~~ARE PO~~ WHERE THE CONCAVITY CHANGED ARE POINTS OF INFLECTION

CONCAVITY RULES

(i) IF $f''(x) > 0$ ON I THEN $f(x)$ IS CONCAVE UP ON I

(ii) IF $f''(x) < 0$ ON I THEN $f(x)$ IS CONCAVE DOWN ON I

$f''(x) > 0$ 

$f''(x) < 0$ 

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DEF'N $x=c$ IS A HYPERCRITICAL NUMBER OF $f(x)$ IF EITHER

(i) $f''(c)=0$

OR (ii) $f''(c)$ IS UNDEFINED.

THE POINT $(c, f(c))$ IS A HYPERCRITICAL POINT IF $x=c$ IS A HYPERCRITICAL NUMBER AND $f(x)$ IS CONTINUOUS AT $x=c$.

POINTS OF INFLECTION CAN ONLY OCCUR AT HYPERCRITICAL POINTS

EX FOR $f(x) = x^4 + 2x^3$ FIND LOCAL EXTREMA, INTERVALS OF CONCAVITY, POINTS OF INFLECTION AND SKETCH ITS GRAPH. (INCLUDE INTERCEPTS).

SOL'N $f'(x) = 4x^3 + 6x^2 = 2x^2(2x+3)$

$f''(x) = 12x^2 + 12x = 12x(x+1)$

SIGN PATTERNS

	$x < -3/2$	$-3/2$	$-3/2 < x < 0$	0	$x > 0$
f'	$-$		$+$		$+$

	$x < -1$	-1	$-1 < x < 0$	0	$x > 0$
f''	$+$		$-$		$+$

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LOCAL EXTREMA LOCAL MIN AT $x = -3/2$

POINTS OF INFLECTION FLEX POINTS AT $x = -1$ AND $x = 0$.

SKETCH INTERCEPTS OF $f(x) = x^4 + 2x^3 = x^3(x+2)$

$(-\frac{3}{2})^3(\frac{1}{2})$

	x	y
INT	-2	0
MIN	$-\frac{3}{2}$	$-\frac{27}{16}$
FLEX	-1	-1
FLEX, INT	0	0