Department of Mathematics and Statistics
Memorial University of Newfoundland

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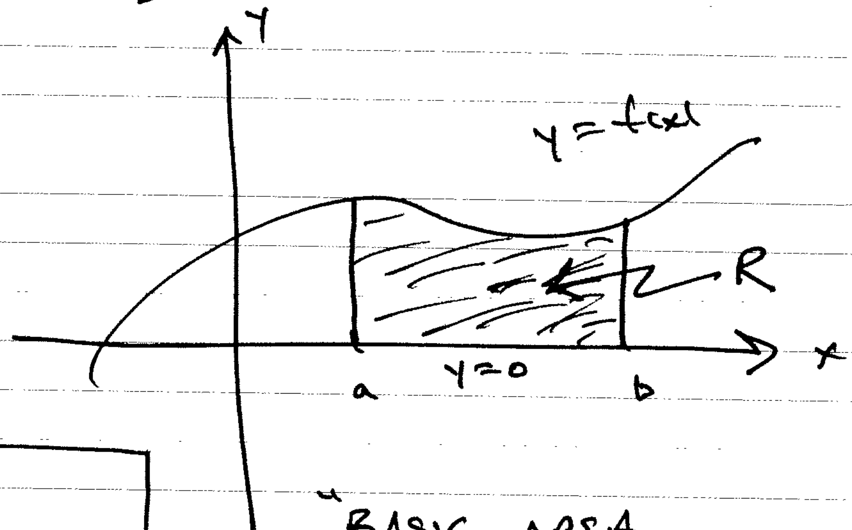
§5.6 APPLICATION OF THE DEFINITE INTEGRAL TO AREA

Suppose $f(x) \geq 0$ for $a \leq x \leq b$

THE REGION R HAS BOUNDARIES

$y = f(x)$	"top"
$y = 0$	"bottom"
$x = a$	left
$x = b$	right

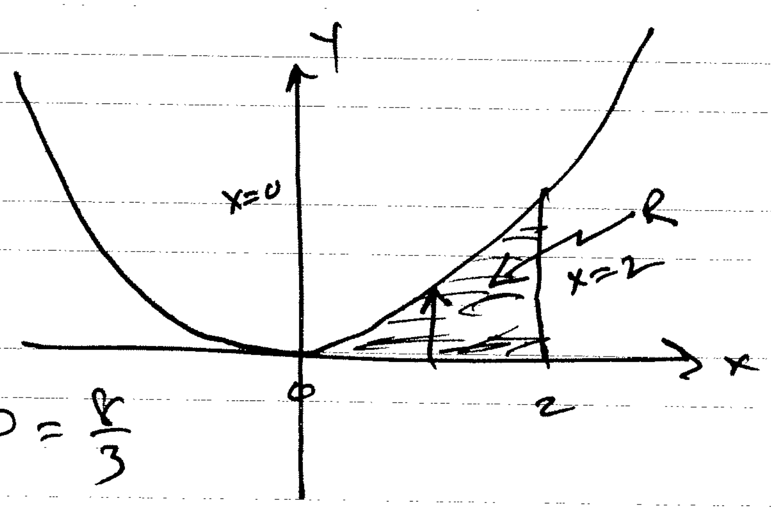


$$\boxed{\text{AREA OF } R = \int_a^b f(x)\,dx}$$

"BASIC AREA FORMULA"

Ex FIND THE AREA ENCLOSED BY $y = x^2$, $y = 0$ AND $x = 2$.

Sol'n



AREA OF R

$$= \int_0^2 x^2\,dx$$

$$= \frac{1}{3}x^3 \Big|_0^2 = \frac{8}{3} - 0 = \frac{8}{3}$$

ENCLOSED AREA IS $\frac{8}{3}$ SQ. UNITS.

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SITUATION #1 X-AXIS IS A BOUNDARY.

CASE 1 X-AXIS IS THE BOTTOM BOUNDARY.

(EXAMPLE #1 IS CASE 1).

CASE 2 X-AXIS IS THE "TOP" BOUNDARY.

EX 2 FIND THE AREA ENCLOSED BY

$y = -\sqrt{x+2}$, $y = 0$, $x = 2$, $x = 7$.

SOL'N

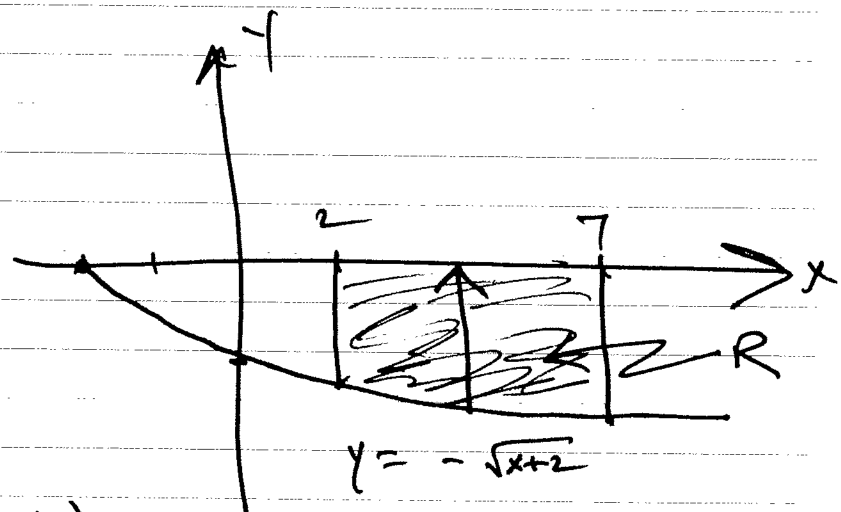
#1 MAKE A **ROUGH** SKETCH

$y = -\sqrt{x+2}$

HALF-PARABOLA

$y^2 = x+2$

or $x = y^2 - 2$



#2 SET UP THE INTEGRAL(S)

$$\text{AREA OF } R = -\int_2^7 -\sqrt{x+2}\,dx$$

$$= \int_2^7 (x+2)^{1/2}\,dx = \frac{2}{3}(x+2)^{3/2}\Big|_2^7 = \left(\frac{2}{3}27\right) - \left(\frac{16}{3}\right)$$

$$= \frac{38}{3}$$ AREA OF R IS $\frac{38}{3}$ SQ UNITS.

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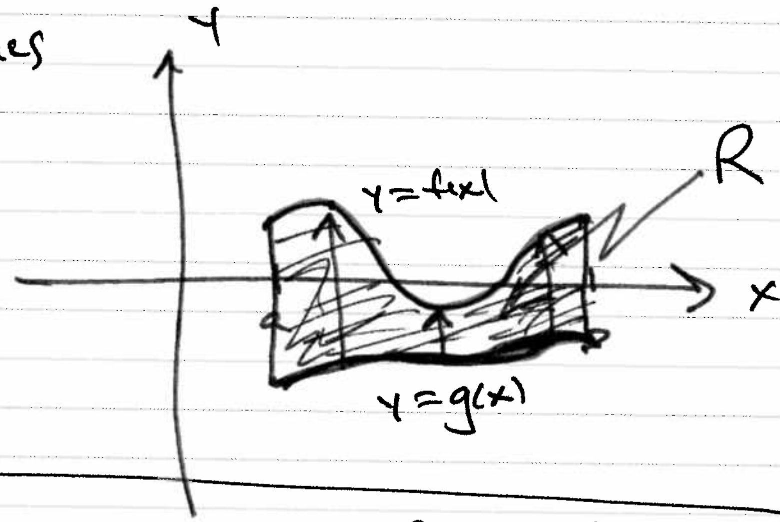
SITUATION 1 X-AXIS IS A BOUNDARY
CASE 1
CASE 2

SITUATION #2 X-AXIS IS NOT A BOUNDARY.

CASE 1 $f(x) \geq g(x)$ FOR $a \leq x \leq b$

REGION, R, HAS BOUNDARIES

(1) $y = f(x)$ "TOP"
(2) $y = g(x)$ "BOTTOM"
(3) $x = a$ left
(4) $x = b$ right



$$\boxed{\text{AREA OF } R = \int_a^b (f(x) - g(x))\,dx \quad \text{. GENERAL AREA FORMULA}}$$

MEMORIZE:
$$\text{AREA} = \int_a^b (\text{TOP CURVE} - \text{BOTTOM CURVE})\,dx$$

EX FIND THE AREA ENCLOSED BY $y = \frac{1}{x}$ AND $y = 3 - 2x$.

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$\boxed{\text{SOLVE } f(x) = g(x) \text{ FOR } x}$

~~PROC~~ PROCEDURE

1) MAKE A ROUGH SKETCH OF BOUNDARY CURVES. FIND ANY RELEVANT POINTS OF INTERSECTION

2) SET UP THE INTEGRAL(S)

3) DO THE CALCULUS

4) SUMMARIZE.

SOL'N POINTS OF INTERSECTION? $y = 1/x$ $y = 3-2x$

SOLVE $\frac{1}{x} = 3 - 2x$ FOR x.

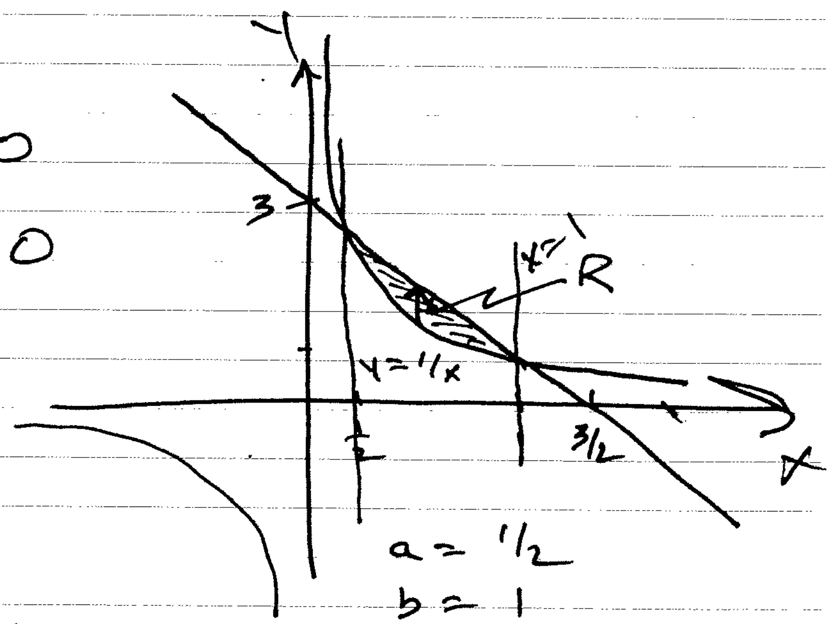
SOLVE $1 = 3x - 2x^2$

SOLVE $2x^2 - 3x + 1 = 0$

SOLVE $(2x-1)(x-1) = 0$

$x = \frac{1}{2}$, $x = 1$.

$(\frac{1}{2}, 2)$, $(1,1)$



$a = 1/2$
$b = 1$

AREA OF R $= \int_{1/2}^{1} (3-2x) - \frac{1}{x}\,dx$

$\boxed{\ln\frac{1}{2} = -\ln 2}$

$= \int_{1/2}^{1} 3 - 2x - \frac{1}{x}\,dx = \left(3x - x^2 - \ln|x|\right)\Big|_{1/2}^{1}$

$= (3 - 1 - 0) - \left(\frac{3}{2} - \frac{1}{4} - \ln\frac{1}{2}\right) = 2 - \frac{5}{4} - \ln 2$

$= \frac{3}{4} - \ln 2.$

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AREA IS $\frac{3}{4} - \ln 2$ SQ. UNITS.

SITUATION #2

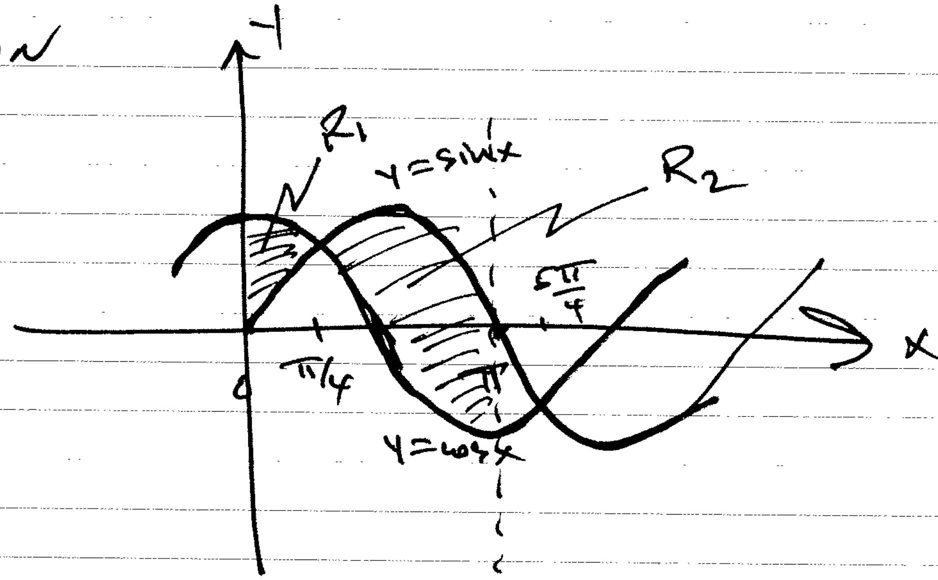
CASE 2 $f(x)$ AND $g(x)$ ARE "INTERTWINED"

EX AREA ENCLOSED BY $y = \sin x$, $y = \cos x$, $x = 0$, $x = \pi$.

POINT OF INTERSECTION

$\sin x = \cos x$

$\tan x = 1$



AREA OF R_1 + AREA OF R_2

$$= \underbrace{\int_0^{\pi/4} \cos x - \sin x \, dx}_{\sqrt{2}-1} + \underbrace{\int_{\pi/4}^{\pi} \sin x - \cos x \, dx}_{1+\sqrt{2}}$$

$= 2\sqrt{2}$ AREA IS $2\sqrt{2}$ SQ. UNITS.