Department of Mathematics and Statistics
Memorial University of Newfoundland

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FTC Justification

FTC comes in two parts.

Suppose we know Part 1 is true and we want to prove Part 2.

For Part 2 we have two anti-derivatives of $f(x)$; namely, $A(x)$ and $F(x)$.

$\therefore$ $A(x) = F(x) + C$ (Theorem 4.10.6)

HENCE,
$$\begin{aligned} \int_a^b f(x)\,dx &= A(b) \\ &= A(b) - A(a) \qquad (A(a) = 0) \\ &= (F(b) + c) - (F(a) + c) \\ &= F(b) - F(a). \end{aligned}$$

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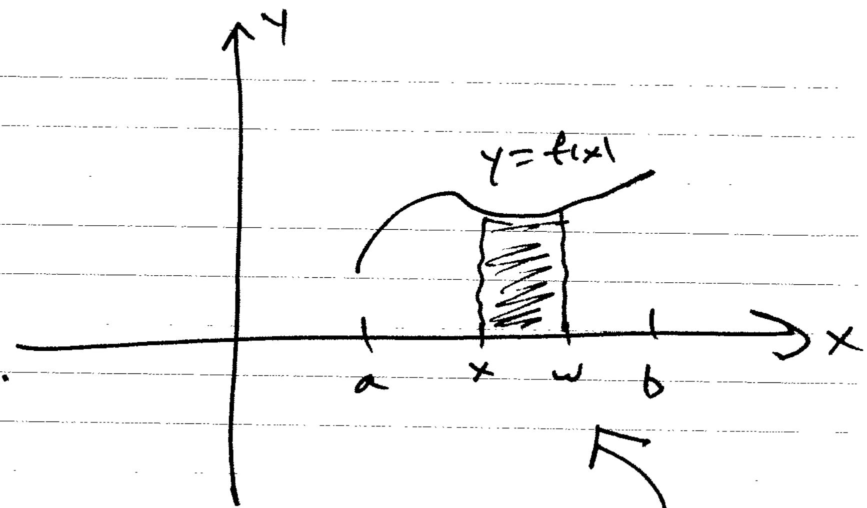
Now, Justify Part 1.

Show:

$A'(x) = f(x)$.

OR,

$$\lim_{w \to x} \frac{A(w) - A(x)}{w - x} = f(x).$$



Suppose $a < x < b$ and $w > x$ as shown

$$A'(x) = \lim_{w \to x} \frac{A(w) - A(x)}{w - x}$$

$$= \lim_{w \to x} \frac{\int_a^w f(t)\,dt - \int_a^x f(t)\,dt}{w - x}$$

$$= \lim_{w \to x} \frac{\int_x^w f(t)\,dt}{w - x}$$

$$= \lim_{w \to x} \frac{\cancel{(w-x)}\,\mathrm{AVE}(f(t), [x, w])}{\cancel{w-x}}$$

$$= \lim_{w \to x} \mathrm{AVE}(f(t), [x\ w])$$

$= f(x)$ SINCE $f(x)$ is CONTINUOUS.