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§5.5 The Fundamental Theorem of The Calculus

Thm Suppose that $f(x) \in \mathcal{C}[a, b]$ and that $F(x)$ is an antiderivative of $f(x)$; that is, that $F'(x) = f(x)$ for $a \leq x \leq b$. Then

$A(x) = F(x) + C$

Part 1: The function $A(x)$ defined, for all x in $[a, b]$, by $A(x) = \int_a^x f(t)\, dt$ is also an antiderivative of $f(x)$.

$A'(x) = f(x)$

Part 2: $\int_a^b f(x)\, dx = F(b) - F(a)$ EVALUATION PART

Use Part 2 to evaluate a definite integral.

Example.

$$\begin{aligned}
\int_1^3 (3x^2 - 4x + 1)\, dx &= (x^3 - 2x^2 + x)\Big|_1^3 \\
&= (27 - 18 + 3) \\
&\qquad -(1 - 2 + 1) \\
&= 12.
\end{aligned}$$

<u>EXAMPLES</u> FIND EACH DEFINITE INTEGRAL.

1. $\int_0^1 x^2\,dx$

<u>SOL'N</u> $\int_0^1 x^2\,dx = \left.\frac{x^3}{3}\right|_0^1$ $\qquad \boxed{F(x)=\frac{1}{3}x^3}$

$= \left(\frac{1}{3}\right) - \left(0\right)$

$= \cancel{F(1)-F(0)}\ \frac{1}{3}$

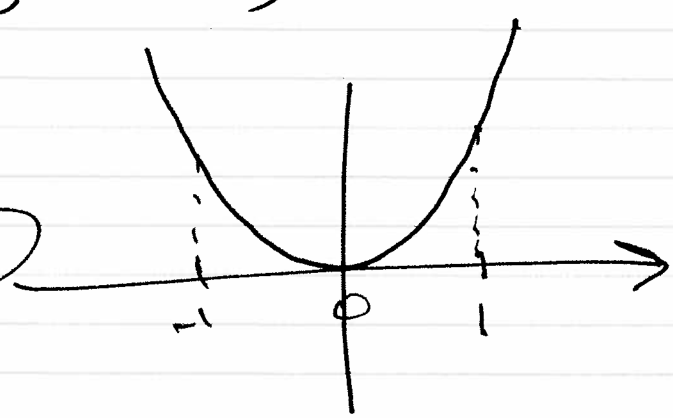
2) $\int_{-1}^{1} x^2\,dx = \left.\frac{1}{3}x^3\right|_{-1}^{1}$

$f(x)=x^2$

$= \left(\frac{1}{3}\right) - \left(\frac{(-1)}{3}\right)$

$= \frac{2}{3}$

$\int_{-1}^{1} f(x)\,dx = 2\int_0^1 f(x)\,dx$ EVEN



$\boxed{f(-x)=f(x)}$

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3) $\displaystyle\int_0^2 \frac{1}{x+1}\,dx$

SOL'N $\displaystyle\int_0^2 \frac{1}{x+1}\,dx$

$= \ln|x+1| \Big|_0^2$

$= \ln 3 - \ln 1$

$= \ln 3$

$\displaystyle\int \frac{1}{x+1}\,dx$

$\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$

$m=1$

$\ln 1 = 0$

METHOD 2 u-substitution for Definite Integrals

SOL'N $\displaystyle\int_0^2 \frac{1}{x+1}\,dx = \int_1^3 \frac{1}{u}\,du$

$\boxed{\begin{aligned} u &= x+1 \\ du &= dx \end{aligned}}$ *

$= \ln|u| \Big|_1^3$

$= \ln 3 - \ln 1$

$= \ln 3$

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#4) $\int_0^2 (2x+1)(x+2)\,dx$

SOL'N TYPE 3

$$\int_0^2 (2x+1)(x+2)\,dx = \int_0^2 (2x^2+5x+2)\,dx \quad \text{TYPE 2}$$

$$= \left(\frac{2}{3}x^3 + \frac{5}{2}x^2 + 2x\right)\Big|_0^2$$

$$= \left(\frac{16}{3} + 10 + 4\right) - (\;0\;)$$

$$= \frac{16}{3} + 14 = \frac{58}{3}$$

(5) $\int_0^{\pi/16} \sin(4x)\,dx$

SOL'N $mx+b = 4x+0$ TYPE I

$$\int_0^{\pi/16} \sin 4x\,dx = \frac{-\cos(4x)}{4}\Big|_0^{\pi/16}$$

$$= \left(-\frac{1}{4}\cos\frac{\pi}{4}\right) - \left(-\frac{\cos 0}{4}\right)$$

$$= -\frac{\sqrt{2}}{8} + \frac{1}{4} = \frac{-\sqrt{2}+2}{8} = \frac{2-\sqrt{2}}{8}$$

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u - SUB

$$\int_0^{\pi/16} \sin 4x \, dx = \int_0^{\pi/4} \sin u \left(\frac{1}{4} du\right)$$

$$\boxed{\begin{aligned} u &= 4x \\ du &= 4\,dx \end{aligned}}$$

$$= \frac{1}{4} \int_0^{\pi/4} \sin u \, du$$

$$= \frac{1}{4} \left(-\cos u\right) \Big|_0^{\pi/4}$$

$$= -\frac{\sqrt{2}}{8} - \left(-\frac{\cos 0}{4}\right) = \frac{2-\sqrt{2}}{8}$$

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(6) $\int_{-1}^{1} e^{-3t} \, dt$ $\qquad$ $\int_{-1}^{1} \frac{1}{e^{3t}} \, dt$

SOL'N

TYPE I

$$\int_{-1}^{1} e^{-3t} \, dt = \frac{e^{-3t}}{-3} \Bigg|_{-1}^{1}$$

$$= \left(-\frac{1}{3} \frac{1}{e^3}\right) - \left(-\frac{1}{3} e^3\right)$$

$$= \frac{1}{3}\left(e^3 - \frac{1}{e^3}\right)$$

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(7) $\int_1^2 \frac{1+x^2}{x}\, dx$

$$\int_a^b f(x)dx = F(b) - F(a)$$

SOL'N TYPE III

$$\int_1^2 \frac{1+x^2}{x}\, dx = \int_1^2 \left(\frac{1}{x} + x\right) dx$$

$$= \left(\ln|x| + \frac{1}{2}x^2\right)\Big|_1^2$$

$$= \left(\ln 2 + \frac{4}{2}\right) - \left(\ln 1 + \frac{1}{2}\right)$$

$$= 2 + \ln 2 - \frac{1}{2} = \frac{3}{2} + \ln 2$$