AMAT3132 – Numerical Analysis, Winter 2010

Home work 5
(show all works)

Due Tuesday March 22, 2010 by 24:00 in the drop box#40
Late assignments are considered as missed work
Full marks 30

Instruction:

- The purpose of this assignment is to understand interpolation.
- Please hand in the report into the box #40 located next to Math general office in HH.
- The first page of the report must include your student information, and a list of people/resources whom (if) you got help from. Examples of people whom you have discussed are your course instructor, your classmates, or any other people. Example of resources are books other than texts for this course, web sites etc.
- You are permitted to discuss each other or people outside the class. However, your matlab code and all results must be authored by yourself.
- Please submit your matlab code electronically using the "submit assignment" utility.
- You must have either one executable **.m** matlab script for the entire assignment - such as the template for this course or you must have one main code that should run and verify your results. If your code does not run, you will be penalized accordingly.
- You may download the divided difference code `divdiff.m` to calculate divided differences.

1. [10]

   (a) Write a MATLAB function for developing a Newton interpolating polynomial $p(x)$ that agrees with the given data set $y_i = f(x_i)$ for $i = 0, 1, \ldots, n$.

   (b) To test your code, consider $y = x^2 e^{-x/2}$, and $h = 0.5, 0.25, 0.125$ to tabulate data in the interval $[-1, 1]$, and determine a $p(x)$ for each h.

   (c) Use $h = 0.01$ to construct interpolating nodes and evaluate $p(x)$ at these nodes in the interval $[-1, 1]$.

   (d) Use the matlab function `interp1` to construct a polynomial $q(x)$ that interpolates your tabulated data to interpolating nodes with $h = 0.01$. Plot $p(x)$, $y(x)$, and $q(x)$ on the same frame of reference.

   (e) Calculate and plot absolute error for $p(x)$ as a function of x for $h = 0.5,\ 0.25$, and 0.125.

2. [10] Repeat the above steps for $y = sin(3\pi x)$, but do not compute $q(x)$.

3. [10] Write a short report summarizing your findings or observations of this assignment. Your report should address, but not limited to, the following:

   (a) How does error behave as a function of h?

   (b) How does error behave as a function of x?

   (c) For a fixed h, do you find that $p(x)$ approximates all functions with the same error bound? Explain.

   (d) What happens if you take $y = sin(6\pi x)$ and compute Newton interpolating polynomial with $h = 0.5, 0.25, 0.125$. Explain why is the bound of error differs with that for interpolating $y = sin(3\pi x)$ at fixed h?