AMAT3132 – Numerical Analysis, Winter 2010

Home work 2
(show all works)

Due Monday Feb 8, 2010 by 24:00 in the drop box#40
Full marks 30

Instruction: (i) Please hand in hard copy of the assignment into the box #40 located next to Math general office in HH.

(ii) Please submit your matlab code in one executable file using the "submit assignment" utility. You must have one executable **.m** file for the entire assignment. (iii) Due date is the same for both code or hard copy.

1. (Analytical question - do not submit)
   Let
   $$A = \begin{bmatrix} 1 & 1+\epsilon \\ 1-\epsilon & 1 \end{bmatrix}$$
   (a) What is the determinant of A?
   (b) In floating point arithmetic, for what range of values of ϵ will the computed value of the determinant be zero?
   (c) What is the LU factorization of A?
   (d) In floating point arithmetic, for what range of values of ϵ will the computed value of U be singular?

2. (Analytical question - do not submit)
   Can Jacobi and Gauss-Seidel methods converge for non-diagonally dominant systems? Explain.

3. (Analytical question - do not submit)
   Consider the linear system:
   $$\begin{aligned} x + z &= 2 \\ -x + y &= 0 \\ x + 2y - 3z &= 0 \end{aligned}$$

(a) Is the coefficient matrix associated to this linear system diagonally dominant?

(b) Do you expect Jacobi and Gauss-Seidel method converge for this system. Explain.

Computer problems

4. [12] Write a MATLAB code to perform the following tasks:

   (a) Write a MATLAB function to calculate the matrix (L_p) norm of a given matrix A. The function should take the matrix A and the norm specification p as input, and should return the norm λ_p.

   Verify your code. The attached table provides useful data for the standard atmosphere. These data are useful to understand the dynamics of the atmosphere, thereby they are key to weather forecasting and climate modeling. Use your matlab function to calculate L_2 and L_∞ norms of p and ρ. Present your result in a tabular form.

   (b) Consider the Hilbert matrix with entries $H_{ij} = \frac{1}{i+j-1}, i, j = 1, \ldots, n$ for any given dimension.

   Let $n = \{25, 50, 75, \ldots, 500\}$. Calculate L_1, L_2, and L_∞ norms of the Hilbert matrix for these given values of n.

   (c) Plot these results on a single graph using linear coordinates.

5. [6]

   (a) Consider the matrix

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 0 & 5 & -2 \\ -1 & -2 & 7 \end{bmatrix}$$

   Using the function you wrote, calculate the L_1, L_2, and L_∞ norms of the Jacobi iteration matrix for A.

   (b) Apply Jacobi method to the system in 3 with different initial guesses. Do the iterations converge for any initial guess? Calculate the spectral radius of the Jacobi iteration matrix.

   (c) Apply Gauss-Seidel method to this system with different initial guesses. Do the iterations converge for any initial guess? Calculate the spectral radius of the Gauss-Seidel iteration matrix.

6. [12] Consider given A and b:

$$A = \begin{bmatrix} 3 & -2 & 0 & 0 & 0 \\ -2 & 5 & 1 & 0 & 0 \\ 0 & 1 & 4 & -2 & 0 \\ 0 & 0 & -2 & 5 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ -9 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

(a) Write down the Jacobi and Gauss-Seidel algorithm for this tri-diagonal system and explain why do you expect a difference in the rate of convergence between these two algoritms.

(b) Solve the system $Ax = b$ using Jacobi and Gauss-Seidel with l_2 norm residual tolerance of `1e-1,1e-2,1e-3,1e-4,1e-5,1e-6,1e-7,1e-8` and record the number of iterations in each case.

(c) Plot the recorded number of iterations as a function of tolerance in the same graph. Explain if these results are consistent with your explanation of rate of convergence in part (6a).

A4. Properties of Standard Atmosphere

The following average values are accepted by international agreement. Here, z is the height above sea level.

z km	T °C	p kPa	ρ kg/m^3
0	15.0	101.3	1.225
0.5	11.5	95.5	1.168
1	8.5	89.9	1.112
2	2.0	79.5	1.007
3	−4.5	70.1	0.909
4	−11.0	61.6	0.819
5	−17.5	54.0	0.736
6	−24.0	47.2	0.660
8	−37.0	35.6	0.525
10	−50.0	26.4	0.413
12	−56.5	19.3	0.311
14	−56.5	14.1	0.226
16	−56.5	10.3	0.165
18	−56.5	7.5	0.120
20	−56.5	5.5	0.088