Amat 3132. HW#3

An $n \times n$ array can be mapped into an $n \times 1$ array according to the following formula.

$$k = j + (i-1)\,n, \quad \text{where}$$

k is the ~~index~~ index in the $n \times 1$ array for (i,j)-th element of the $n \times n$ array. In MatLab, one get

```
function [k] = points(i, j, n)

        k = j + (i-1)* n
```

For $n=5$, the mapping is listed below

k = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

(i, j) = (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5)

k = 11, 12, 13, 14, 15, 16, 17, 18, 19, 20,

(i, j) = (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2), (4,3), (4,4), (4,5)

k = 21, 22, 23, 24, 25.

(i, j) = (5,1), (5,2), (5,3), (5,4), (5,5)

#2

$$(Ax)_k = 4u_{ij} - u_{i-1\,j} - u_{i+1\,j} - u_{i\,j+1} - u_{i\,j-1}$$

$$i=1 \begin{cases} j=1, & 4u_{11} - u_{12} - u_{21} \\ j=2, & 4u_{12} - u_{13} - u_{11} - u_{22} \\ j=3, & 4u_{13} - u_{14} - u_{12} - u_{23} \end{cases}$$

Using $k = j + (i-1)n$, we get the following

$$\begin{array}{c|cccccccc} & u_{11} & u_{12} & u_{13} & \cdots\cdots & u_{21} & u_{22} & \cdots\cdots \\ (1,1) & 4 & -1 & & & -1 & & \\ (1,2) & -1 & 4 & -1 & & & -1 & \\ (1,3) & & -1 & 4 & -1 & & & -1 \end{array}$$

Let $t_{ij} = \begin{cases} 4 & \text{if } i=j \\ -1 & \text{if } i-1=j \\ -1 & \text{if } i+1=j \end{cases}$

$T = [t_{ij}]_{n \times n}$ is a tri-diagonal matrix.

Let $D = \begin{bmatrix} -1 & 0 & \cdots & \\ 0 & -1 & \cdots & \\ \cdots & & & \end{bmatrix}_{n \times n}$

be a diagonal matrix.

First row of A is

$[T \; D \cdots \emptyset \; 0 \cdots \;]$

Similarly, 2nd row of A is

$[D \; T \; D \cdots \emptyset \; 0 \cdots \;]$

Therefore.

$$A = \begin{bmatrix} T & D & \phi & \cdots & & \\ D & T & D & \phi & & \\ \phi & D & T & D & \phi & \\ & & & & & \end{bmatrix}$$

Size of A is $n^2 \times n^2$.

A is a blocked tri-diagonal sparse matrix

The matrix is diagonally dominant.

For $n=3$

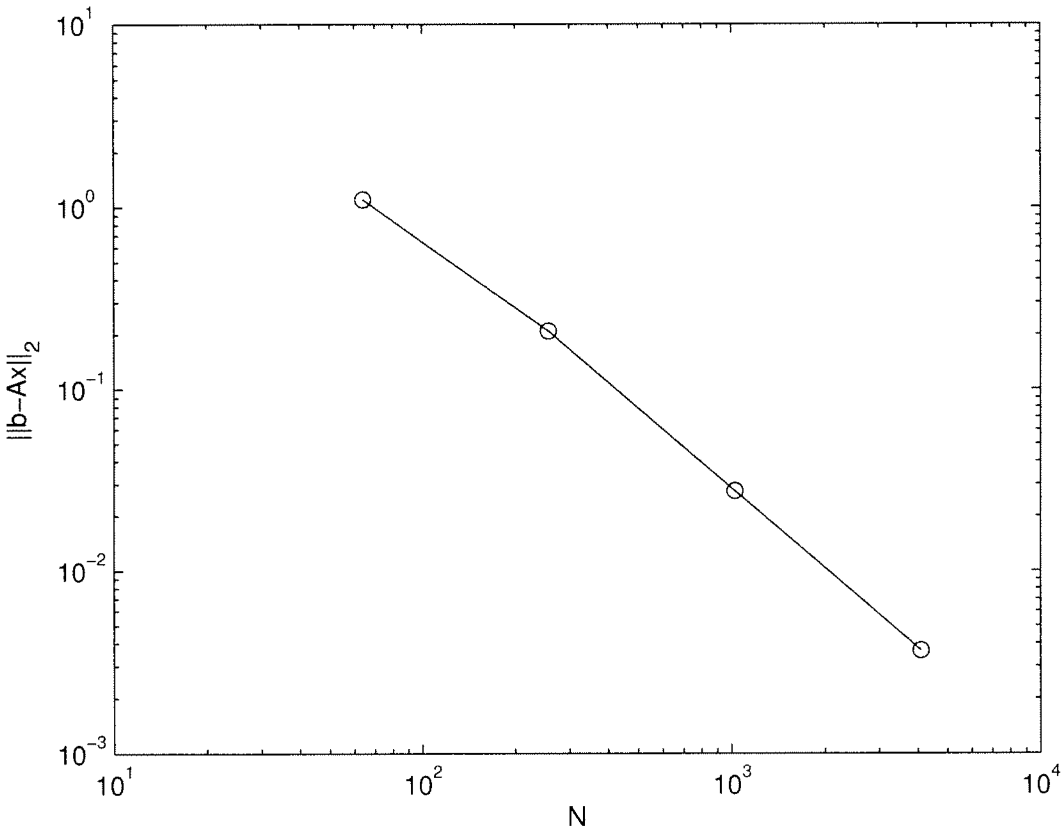
$$A=\left[\begin{array}{ccc|ccc|ccc}
4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 4 & -1 & 0 & -1 & 0 & & \phi & \\
0 & -1 & 4 & 0 & 0 & -1 & & & \\
\hline
 & & & & & & & & \\
\hline
 & \phi & & & \cdots & 0 & -1 & 0 \;\; -1 & 4
\end{array}\right]$$

For $n=6$

$$T=\begin{bmatrix} 4 & -1 & & \\ -1 & 4 & -1 & \\ & \ddots & \ddots & \ddots \\ & & & 4 \end{bmatrix} \quad 6\times 6 \text{ matrix.}$$

$$D=\begin{bmatrix} -1 & 0 & \cdots \\ 0 & -1 & \cdots \\ & & \end{bmatrix} \quad 6\times 6 \text{ matrix.}$$

$$A=\begin{bmatrix} T & D & \phi & \cdots \\ D & T & D & \phi & \cdots \\ \phi & D & T & D \\ \cdots & & & \end{bmatrix} \quad 36\times 36 \text{ matrix.}$$



10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
$\|b-Ax\|_2$
10^{1}
10^{2}
10^{3}
10^{4}
N