# AMAT3132 – Numerical Analysis, Winter 2010

Home work 2 **Solution**
(show all works)

Due Monday Feb 8, 2010 by 24:00 in the drop box#40
Full marks 30

4(a)

The developed function calculates norms of matrices and vectors.

| | | p | e |
|---|---|---|---|
| $L_2$ | this code | 224.42 | 2.87 |
| | matlab | 224.42 | 2.87 |
| $L_\infty$ | this code | 101.30 | 1.23 |
| | matlab | 101.30 | 1.23 |

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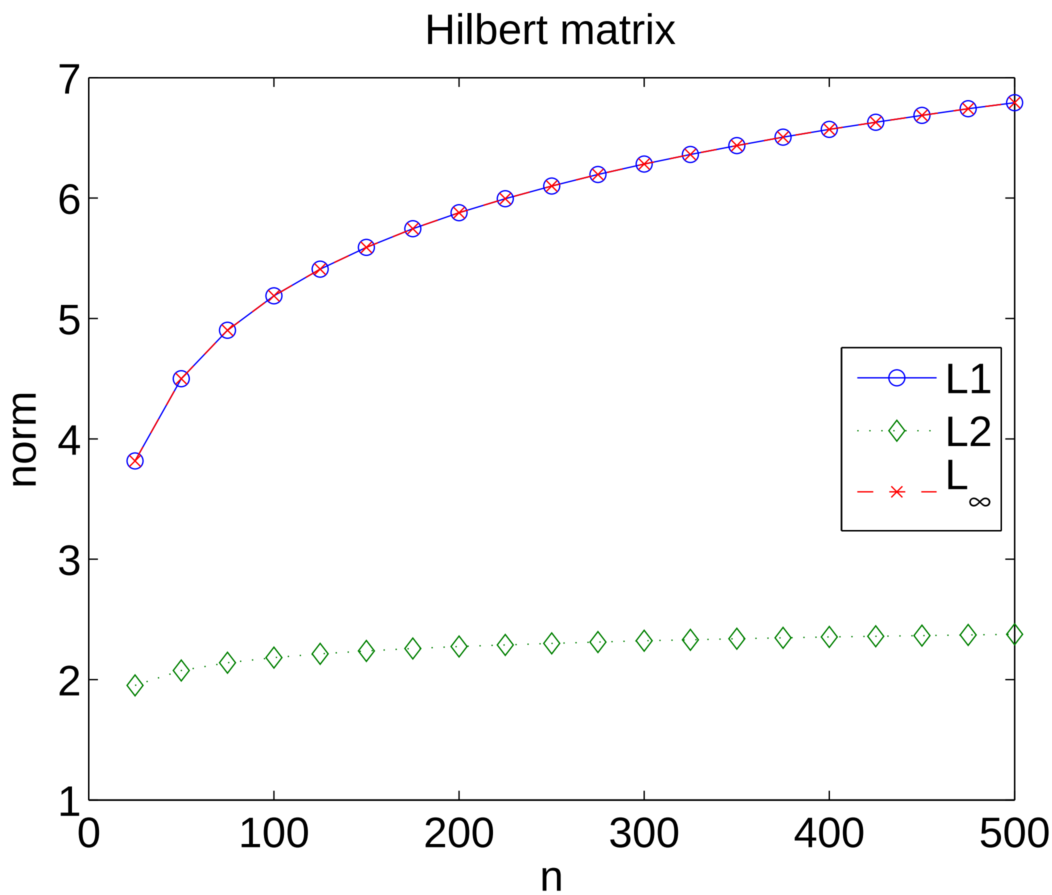
4(c) $L_1$, $L_2$, and $L_\infty$ norms of Hilbert matrix is shown in Fig 1.

"If $H$ is a Hilbert matrix then $H^T = H$. Therefore, we see that $L_1$ and $L_\infty$ noms are same in Fig. 1."
(This Remark is not necessary to get credit for this question.).

4(b)

result = matrix_norm(A, p) is the function available in the attached cod.



Hilbert matrix
norm
n
L1
L2
$L_\infty$
0
100
200
300
400
500
1
2
3
4
5
6
7

5(a) $$A = \begin{bmatrix} 4 & 3 & 2 \\ 0 & 5 & -2 \\ -1 & -2 & 7 \end{bmatrix}$$

Jacobi iteration matrix

$$J = -D^{-1}(L+U)$$

is calculated using the developed function " result = Jac_mat (A) " and norms are calculated using "matrix_norm(A,P)"

$L_1(J) = 1.4 \times 10^{-1}$

$L_2(J) = 3.9 \times 10^{-1}$

$L_\infty(J) = 4.2 \times 10^{-1}$

—o—

5(b) The system is

$$x + z = 2$$

$$-x + y = 0$$

$$x + 2y - 3z = 0$$

| Initial guess | # of iteration |
|---|---|
| $[0, 0, 0]^T$ | 216 |
| $[10, 0, 0]^T$ | 244 |
| $[0, 10, 0]^T$ | 235 |

(Students may choose any initial guess but should observe high # of iterations)
# of iterations > 200 for solving a 3X3 system is "too" high. ("How high" is high enough ?")

spectral radius of J $= 9.4 \times 10^{-1}$ which smaller than 1. Thus Jacobi method converges. Note $\rho(J)$ is close to 1, which implies slow convergence.

5(C) Gauss-Seidel method.

| Ini. Guess. | # of iterations | Res. |
|---|---|---|
| $[0,0,0]^T$ | 500 | Res. > tol $(=10^{-5})$ |
| $[10,0,0]^T$ | 500 | " " |
| $[0,10,0]^T$ | 500 | " " |

Gauss-Seidel method does not converge.

Iteration matrix is

$G = -(L+D)^{-1}U$, which is computed by function Gas_mat(A).

$\rho(G) = 1$ !

Convergence criterion is not satisfied.

6(a) Jacobi iteration for solving

$Ax = b$ is

$$x^{(k+1)} = x^k + D^{-1}(b - Ax^k) \qquad (1)$$

Gauss-Seidel iteration is defined by

$$x^{(k+1)} = x^k + (L+D^{-1})(b - Ax^k) \qquad (2)$$

The scheme (2) uses approximated solutions as soon as they are available.

The scheme (1) waits until all components of $x^{(k)}$ have been approximated before using newly computed values.

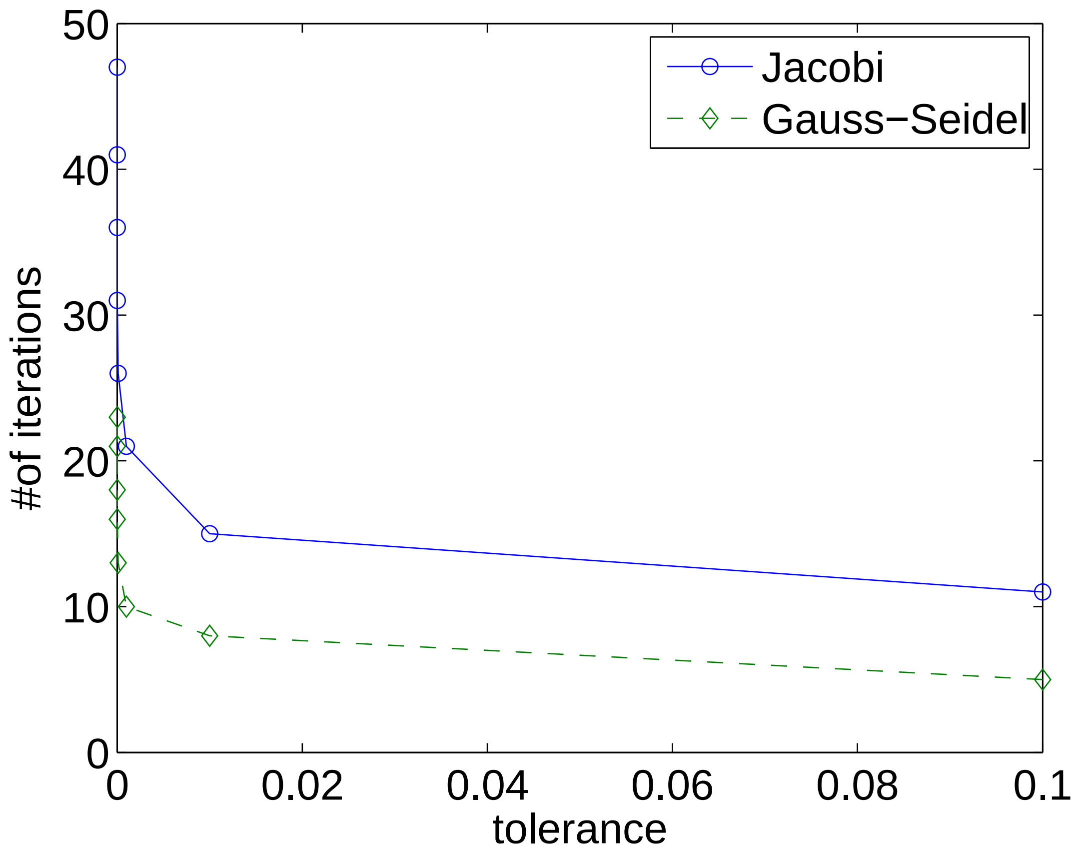
That's why we expect that scheme (2) could converges faster than (1)

6(b)

| tol. | # of iteration: Jacobi | # of iteration: Gauss-Seidel. |
|---|---|---|
| $10^{-1}$ | 11 | 5 |
| $10^{-2}$ | 15 | 8 |
| $10^{-3}$ | 21 | 10 |
| $10^{-4}$ | 26 | 13 |
| $10^{-5}$ | 31 | 16 |
| $10^{-6}$ | 36 | 18 |
| $10^{-7}$ | 41 | 21 |
| $10^{-8}$ | 47 | 23 |

6(c) Plot is attached. Clearly, Gauss-Seidel method takes less # iterations than Jacobi - as expected.

As a rough estimate the speed up is about a factor of 2.



Jacobi
Gauss−Seidel
#of iterations
tolerance
0
10
20
30
40
50
0
0.02
0.04
0.06
0.08
0.1