MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.4 | **Math 2050 Worksheet** | WINTER 2018

For practice only. Not to be submitted.

1. Solve each of the following homogeneous systems of equations using Gaussian elimination and back-substitution. If a solution exists, express it as a vector or as a linear combination of vectors.

   (a) $$\left.\begin{array}{rcrcrcl} a & - & 2b & - & 2c & = & 0 \\ -4a & + & 8b & + & 6c & = & 0 \end{array}\right\}$$

   (b) $$\left.\begin{array}{rcrcrcrcl} x_1 & - & 3x_2 & & & + & 4x_4 & = & 0 \\ -x_1 & + & x_2 & + & 4x_3 & - & 2x_4 & = & 0 \\ x_1 & & & - & 6x_3 & + & x_4 & = & 0 \\ 2x_1 & - & 5x_2 & - & 2x_3 & + & 7x_4 & = & 0 \end{array}\right\}$$

2. Using your answers to #1, show that solutions to the following systems of equations can be written in the form $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_h$, where $\mathbf{x}_p$ is a particular solution of the given system and $\mathbf{x}_h$ is a solution of the corresponding homogeneous system.

   (a) $$\left.\begin{array}{rcrcrcr} a & - & 2b & - & 2c & = & -5 \\ -4a & + & 8b & + & 6c & = & 9 \end{array}\right\}$$

   (b) $$\left.\begin{array}{rcrcrcrcl} x_1 & - & 3x_2 & & & + & 4x_4 & = & 6 \\ -x_1 & + & x_2 & + & 4x_3 & - & 2x_4 & = & -8 \\ x_1 & & & - & 6x_3 & + & x_4 & = & 9 \\ 2x_1 & - & 5x_2 & - & 2x_3 & + & 7x_4 & = & 13 \end{array}\right\}$$

3. Use Gaussian elimination and back-substitution to determine whether each of the following sets of vectors is linearly independent or linearly dependent.

   (a) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ -4 \\ -1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -3 \\ -1 \end{bmatrix}$

   (b) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 7 \\ -5 \\ -6 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 5 \\ -1 \\ 0 \\ 3 \end{bmatrix}$

   (c) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 3 \\ 4 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 6 \\ 4 \\ 0 \\ -4 \end{bmatrix}$

PLEASE TURN OVER

4. Prove that if A and B are suitably-sized matrices with linearly independent columns then their product AB also has linearly independent columns.