MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 1.1 **Math 2050 Worksheet** WINTER 2018

For practice only. Not to be submitted.

1. Given a point $B = (4, -4)$, find the point A and sketch the vector $\overrightarrow{AB}$ such that

   (a) $\overrightarrow{AB} = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$ (b) $\overrightarrow{AB} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

   (c) $\overrightarrow{AB} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ (d) $\overrightarrow{AB} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$

2. Simplify each of the following vectors.

   (a) $\begin{bmatrix} 4 \\ 1 \end{bmatrix} + 3\begin{bmatrix} -2 \\ 0 \end{bmatrix} - \begin{bmatrix} -6 \\ 7 \end{bmatrix}$ (b) $6\begin{bmatrix} -3 \\ 3 \\ -7 \end{bmatrix} - 4\begin{bmatrix} 0 \\ 5 \\ -1 \end{bmatrix}$

   (c) $-k\begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix} + 3\begin{bmatrix} k \\ 1 \\ -2 \end{bmatrix}$

3. Determine whether $\begin{bmatrix} 1 \\ 6 \end{bmatrix}$ can be expressed as a linear combination of

   (a) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 9 \end{bmatrix}$ (b) $\begin{bmatrix} 6 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 20 \\ -8 \end{bmatrix}$

4. Determine whether $\begin{bmatrix} -7 \\ 0 \\ 6 \end{bmatrix}$ can be expressed as a linear combination of

   (a) $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ -4 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 7 \\ 3 \end{bmatrix}$

5. (a) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors. Show that if $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ then it must also be a linear combination of $5\mathbf{u}$ and $-\mathbf{v}$.

   (b) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors. Show that if $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ then it must also be a linear combination of $k\mathbf{u}$ and $\ell\mathbf{v}$ for any non-zero scalars k and ℓ.

6. Prove that vectors $\mathbf{u}$ and $\mathbf{v}$ are parallel *if and only if* there is a non-trivial linear combination of $\mathbf{u}$ and $\mathbf{v}$ equal to the zero vector $\mathbf{0}$. (By a "non-trivial" linear combination, we mean that there exist scalars a and b *not equal to zero* such that $a\mathbf{u} + b\mathbf{v} = \mathbf{0}$.) In other words, prove each of the following:

   (a) If $\mathbf{u}$ and $\mathbf{v}$ are parallel, then there is a non-trivial combination of $\mathbf{u}$ and $\mathbf{v}$ equal to $\mathbf{0}$.

   (b) If there is a non-trivial combination of $\mathbf{u}$ and $\mathbf{v}$ equal to $\mathbf{0}$, then $\mathbf{u}$ and $\mathbf{v}$ are parallel.