A Primer on Applications of Vectors

The theory of vectors studied in Unit 1 of Math 2050 lends itself to several types of applications. However, many of these are of a very similar nature, which makes them easy to confuse. Here's a brief "cheat sheet" outlining some prominent applications, and the typical solution method. (Be warned that this is *not* an exhaustive list, and sometimes an easier solution may present itself!)

Note that in the following, the word "line" can often be replaced by the word "vector," specifically meaning the direction vector of the line.

- **Intersection of two lines**: Find the vector equations of the two lines. Be sure to differentiate between the parameters (use t for one line and s for the other, for example). Then set the two vector equations equal to each other. Looking at each component, this gives three equations in two unknowns (t and s). Solve for t and s using two of these equations, then substitute the solution into the third equation to ensure that it works there as well. If values of t and s can be found, substitute one or the other back into the appropriate line equation to retrieve the point of intersection. If no such values of t and s can be found, the two lines do not intersect.

- **Intersection of a line and a plane**: From the vector equation of the line, obtain the equations for x, y and z (in terms of the parameter t). Substitute these into the equation of the plane and solve for t. If a value of t results, substitute it back into the vector equation of the line to obtain the point of intersection. If no such value of t exists, the line and plane do not intersect.

- **Locating two orthogonal vectors in a plane**: Solve for one of x, y or z from the equation of the plane. Substitute this into a general vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and rewrite it as a linear combination of two vectors (call them $\mathbf{u}$ and $\mathbf{v}$). These vectors must lie in the plane. If their dot product is 0, they are orthogonal. If not, project one vector onto the other — say, $\mathbf{u}$ onto $\mathbf{v}$ — to obtain $\mathbf{p}$. Then $\mathbf{u} - \mathbf{p}$ will be orthogonal to $\mathbf{v}$.

- **Distance from a point P to a line**: Pick any point Q on the line (by substituting some value of t, usually $t = 0$, into the vector equation for the line). Calculate the vector $\mathbf{u} = \overrightarrow{QP}$ which starts at Q and ends at P. Project $\mathbf{u}$ onto the direction vector of the line to get $\mathbf{p}$. Then $\|\mathbf{u} - \mathbf{p}\|$ is the distance from P to the line.

- **Distance from a point P to a plane**: Pick any point Q in the plane (by choosing values for two of x, y and z, substituting these into the equation of the plane, and solving for the remaining variable). Calculate the vector $\mathbf{u} = \overrightarrow{PQ}$ which starts at P and ends at Q. Project $\mathbf{u}$ onto the normal to the plane to get $\mathbf{p}$. Then $\|\mathbf{p}\|$ is the distance from P to the plane.