MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2 — **MATH 2050** — MARCH 21ST, 2018

Name	**MUN Number**

[6] 1. (a) Solve the following homogeneous system using Gaussian elimination. If a solution exists, express it as a linear combination of vectors.

$$\left.\begin{array}{rcrcrcl} 3x & - & y & - & 2z & = & 0 \\ -x & + & 2y & - & 6z & = & 0 \\ x & - & y & + & 2z & = & 0 \end{array}\right\}$$

[2] (b) Given that $\mathbf{x}_p = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ is a particular solution to the system

$$\left.\begin{array}{rcrcrcl} 3x & - & y & - & 2z & = & 7 \\ -x & + & 2y & - & 6z & = & 6 \\ x & - & y & + & 2z & = & -1 \end{array}\right\}$$

use your answer to part (a) to write a general solution as a sum of $\mathbf{x}_p$ and $\mathbf{x}_h$, the solution to the corresponding homogeneous system.

[2] (c) Using your answer to part (a), deduce whether the vectors $\mathbf{v}_1 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} -2 \\ -6 \\ 2 \end{bmatrix}$ are linearly independent or linearly dependent. Explain your answer.

[6] 2. Find conditions on k such that the following system has <u>no solutions</u>.

$$\left.\begin{array}{rcrcl} x & + & y & = & 0 \\ 2x & + & 3y & = & k \\ -4x & + & ky & = & 5 \end{array}\right\}$$

[12] 3. Let $A = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 & -2 \\ 5 & 0 & 3 \end{bmatrix}$. Evaluate each of the following, if possible:

$$A + B, \quad A + A^T, \quad AB, \quad BA, \quad A^2, \quad B^2.$$

If an expression cannot be evaluated, explain why not.

[6] 4. (a) Use Gaussian elimination to find the inverse of the matrix $A = \begin{bmatrix} 0 & -5 & 0 \\ 1 & 0 & -3 \\ 2 & 0 & -5 \end{bmatrix}$.

[6] (b) Write A as a product of elementary matrices.

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