MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 1 | **MATH 2050** | February 14th, 2018

Name | **MUN Number**

1. Consider the vectors $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 1 \\ 6 \\ -2 \end{bmatrix}$.

[3] (a) Give a unit vector in the opposite direction to $\mathbf{u}$.

[3] (b) Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$. (You do not need to find the angle itself.)

[5] (c) Determine whether $\mathbf{w}$ lies in the plane spanned by $\mathbf{u}$ and $\mathbf{v}$. (You do not need to find the equation of this plane.)

[3] (d) Find a normal to the plane spanned by $\mathbf{v}$ and $\mathbf{w}$.

2. Let π be the plane with equation $x + y - 2z = 4$.

[3] (a) Give the vector equation of the line ℓ perpendicular to π which passes through the point $(1, 3, 6)$.

[5] (b) Find the point of intersection Q of the plane π and the line ℓ.

[6] (c) Find the distance from the point $P(-4, 0, 2)$ to π.

[6] 3. Determine whether the vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ -5 \\ -4 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -3 \\ -5 \\ 0 \\ 8 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -1 \\ -1 \\ 2 \\ 0 \end{bmatrix}$$

are linearly independent or linearly dependent. If they are linearly dependent, express $\mathbf{w}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

[6] 4. Consider two vectors $\mathbf{u}$ and $\mathbf{v}$.

(a) Define what it means for $\mathbf{u}$ and $\mathbf{v}$ to be **orthogonal**.

(b) Define what it means for $\mathbf{u}$ and $\mathbf{v}$ to be **linearly independent**.

(c) Suppose $\mathbf{u}$ and $\mathbf{v}$ are orthogonal non-zero vectors. Prove that $\mathbf{u}$ and $\mathbf{v}$ must be linearly independent.

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