MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.3 **Math 2000 Worksheet** WINTER 2018

SOLUTIONS

1. (a) $z + w = (3 - 4i) + (-1 + 7i) = (3 - 1) + (-4 + 7)i = 2 + 3i$

   (b) $z - w = (3 - 4i) - (-1 + 7i) = (3 + 1) + (-4 - 7)i = 4 - 11i$

   (c) $z \cdot w = (3 - 4i) \cdot (-1 + 7i) = -3 + 4i + 21i - 28i^2 = (-3 + 28) + (4 + 21)i = 25 + 25i$

   (d) $\dfrac{w}{z} = \dfrac{-1 + 7i}{3 - 4i} \cdot \dfrac{3 + 4i}{3 + 4i} = \dfrac{-3 + 21i - 4i + 28i^2}{9 - 16i^2} = \dfrac{(-3 - 28) + (21 - 4)i}{9 + 16} = \dfrac{31}{25} - \dfrac{17}{25}i$

   (e) $w^2 = (-1 + 7i)^2 = 1 - 7i - 7i + 49i^2 = (1 - 49) + (-7 - 7)i = -48 - 14i$

   (f) $|z| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$

2. (a) Recalling that we need to use the complex dot product, we have

   $$\mathbf{v} \cdot \mathbf{w} = (-3i)(-2i) + 7(5i) + (4 - i)(1 - 6i) = 6i^2 + 35i + 4 - 24i - i + 6i^2 = -8 + 10i.$$

   (b) First observe that

   $$\mathbf{v} \cdot \mathbf{v} = (-3i)(3i) + 7(7) + (4 - i)(4 + i) = -9i^2 + 49 + 16 - i^2 = 75$$

   so

   $$\|\mathbf{v}\| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{75} = 5\sqrt{3}.$$

   (c) From part (b), such a unit vector is

   $$\frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{5\sqrt{3}} \begin{bmatrix} 3i \\ 7 \\ 4 + i \end{bmatrix} = \frac{\sqrt{3}}{15} \begin{bmatrix} 2i \\ 7 \\ 4 + i \end{bmatrix}.$$