MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.2 **Math 2050 Worksheet** WINTER 2018

SOLUTIONS

1. First observe that

$$\det(A^3B^T(-2B)A^{-1}) = \det(A^3)\det(B^T)\det(-2B)\det(A^{-1}).$$

First, $\det(A^3) = [\det(A)]^3 = (-7)^3 = -343$. Second, $\det(B^T) = \det B = 3$. Third, $\det(-2B) = (-2)^5 \det B = (-32)(3) = -96$. Finally, $\det(A^{-1}) = \frac{1}{\det A} = \frac{1}{7}$. Therefore,

$$\det(A^3B^T(-2B)A^{-1}) = (-343)(3)(-96)\left(\frac{1}{-7}\right) = -14112.$$

2. First we reduce A to row-echelon form, keeping track of row interchanges and row multiplications. We have

$$\begin{bmatrix} 0 & 4 & -2 & 6 \\ 1 & 3 & 0 & -2 \\ 1 & 0 & 1 & -2 \\ -1 & 1 & -4 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 3 & 0 & -2 \\ 0 & 4 & -2 & 6 \\ 1 & 0 & 1 & -2 \\ -1 & 1 & -4 & 0 \end{bmatrix}$$

$$\xrightarrow[R_4 \to R_4 - (-1)R_1]{R_3 \to R_3 - R_1} \begin{bmatrix} 1 & 3 & 0 & -2 \\ 0 & 4 & -2 & 6 \\ 0 & -3 & 1 & 0 \\ 0 & 4 & -4 & -2 \end{bmatrix} \xrightarrow{R_2 \to \frac{1}{4}R_2} \begin{bmatrix} 1 & 3 & 0 & -2 \\ 0 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & -3 & 1 & 0 \\ 0 & 4 & -4 & -2 \end{bmatrix}$$

$$\xrightarrow[R_4 \to R_4 - 4R_2]{R_3 \to R_3 - (-3)R_2} \begin{bmatrix} 1 & 3 & 0 & -2 \\ 0 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{9}{2} \\ 0 & 0 & -2 & -8 \end{bmatrix} \xrightarrow{R_3 \to (-2)R_3} \begin{bmatrix} 1 & 3 & 0 & -2 \\ 0 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -9 \\ 0 & 0 & -2 & -8 \end{bmatrix}$$

$$\xrightarrow{R_4 \to R_4 - (-2)R_3} \begin{bmatrix} 1 & 3 & 0 & -2 \\ 0 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & -26 \end{bmatrix}.$$

The determinant of this final matrix is $(1)(1)(1)(-26) = -26$. We have performed one row interchange, and performed scalar multiplications of rows by $\frac{1}{4}$ and -2. Hence

$$\det A = (-1)(4)\left(-\frac{1}{2}\right)(-26) = -52.$$