MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2 **MATH 2050** WINTER 2018

SOLUTIONS

[6] 1. (a) We can write this system in the form $A\mathbf{x} = \mathbf{0}$ where

$$A = \begin{bmatrix} 3 & -1 & -2 \\ -1 & 2 & -6 \\ 1 & -1 & 2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -6 \\ 3 & -1 & -2 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - (-1)R_1} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -4 \\ 0 & 2 & -8 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

so then $z = t$ is a free variable, $y = 4z = 4t$ and $x = y - 2z = 2t$. Hence

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2t \\ 4t \\ t \end{bmatrix} = t \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}.$$

[2] (b) Since $\mathbf{x}_h = t \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$, we have

$$\mathbf{x} = \mathbf{x}_p + \mathbf{x}_h = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} + t \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}.$$

[2] (c) The columns of the matrix A are exactly the vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$, and the solution to the equation $A\mathbf{x} = \mathbf{0}$ has an infinite number of solutions, which means that there are non-trivial linear combinations such that

$$x\mathbf{v}_1 + y\mathbf{v}_2 + z\mathbf{v}_3 = \mathbf{0}.$$

Hence these vectors are linearly <u>dependent</u>.

[6] 2. We row-reduce the augmented matrix:

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 3 & k \\ -4 & k & 5 \end{array}\right] \xrightarrow[R_3 \to R_3 - (-4)R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & k \\ 0 & k+4 & 5 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - (k+4)R_2} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & k \\ 0 & 0 & 5 - k(k+4) \end{array}\right].$$

This system will only have solutions if

$$\begin{aligned} 5 - k(k+4) &= 0 \\ k^2 + 4k - 5 &= 0 \\ (k+5)(k-1) &= 0, \end{aligned}$$

that is, if $k = -5$ or $k = 1$. Thus the system has no solutions if $k \neq -5$ and $k \neq 1$.

[12] 3.
- $A + B$ cannot be evaluated because A and B are not of the same size
- $A + A^T = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -1 \\ -1 & 4 \end{bmatrix}$
- $AB = \begin{bmatrix} -1 & -12 & -11 \\ 10 & 0 & 6 \end{bmatrix}$
- BA cannot be evaluated because B has 3 columns, but A has only 2 rows
- $A^2 = \begin{bmatrix} 16 & -6 \\ 0 & 4 \end{bmatrix}$
- B^2 cannot be evaluated because B is not a square matrix

[6] 4. (a) We have

$$\left[\begin{array}{ccc|ccc} 0 & -5 & 0 & 1 & 0 & 0 \\ 1 & 0 & -3 & 0 & 1 & 0 \\ 2 & 0 & -5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 1 & 0 \\ 0 & -5 & 0 & 1 & 0 & 0 \\ 2 & 0 & -5 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 1 & 0 \\ 0 & -5 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to -\frac{1}{5}R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 1 & 0 \\ 0 & 1 & 0 & -\frac{1}{5} & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - (-3)R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -5 & 3 \\ 0 & 1 & 0 & -\frac{1}{5} & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right]$$

and so

$$A^{-1} = \begin{bmatrix} 0 & -5 & 3 \\ -\frac{1}{5} & 0 & 0 \\ 0 & -2 & 1 \end{bmatrix}.$$

[6] (b) The elementary matrices corresponding to the row operations in part (a) are

$$E_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The inverses of these matrices are

$$E_1^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}, \quad E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_4^{-1} = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Thus

$$\begin{aligned} A &= E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned}$$