MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATHICS AND STATISTICS

Assignment 2 | **MATH 2050** | Winter 2018

SOLUTIONS

[4] 1. If the plane is perpendicular to the line ℓ then the direction vector for ℓ,

$$\mathbf{d} = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix},$$

is a normal to the plane. Thus the equation of the plane has the form

$$3x + 4y - z = d.$$

Since $P(1, -2, -9)$ is a point in the plane,

$$d = 3(1) + 4(-2) - (-9) = 4,$$

so the equation of the plane is

$$3x + 4y - z = 4.$$

[6] 2. We need two vectors in the plane, such as

$$\overrightarrow{BA} = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} \quad \text{and} \quad \overrightarrow{CA} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}.$$

A normal to the plane is

$$\mathbf{n} = \overrightarrow{BA} \times \overrightarrow{CA} = \begin{vmatrix} 2 & 1 \\ 4 & 4 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 5 & 1 \\ 0 & 4 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 5 & 2 \\ 0 & 4 \end{vmatrix} \mathbf{k} = 4\mathbf{i} - 20\mathbf{j} + 20\mathbf{k} = \begin{bmatrix} 4 \\ -20 \\ 20 \end{bmatrix}.$$

Thus the equation of the plane has the form

$$4x - 20y + 20z = d.$$

Since $A(3, 0, 1)$ is a point in the plane,

$$d = 4(3) - 20(0) + 20(1) = 32,$$

so the equation of the plane is

$$4x - 20y + 20z = 32.$$

[4] 3. A direction vector for the line is

$$\mathbf{d} = \overrightarrow{AB} = \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$$

so an appropriate vector equation is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}.$$

The corresponding parametric equations are

$$x = -7 + 5t$$
$$y = 1 + 4t$$
$$z = 2 + 3t.$$

[6] 4. (a) We will use s the represent the parameter of the second line. Then, comparing the corresponding parametric equations, we must have

$$8 - 6t = -s$$
$$-5 + t = 11 + 3s$$
$$2 + 4t = -6 - 2s.$$

Multiplying the second equation by 6 and adding it to the first equation gives

$$-22 = 66 + 17s \quad \Longrightarrow \quad 17s = -88 \quad \Longrightarrow \quad s = -\frac{88}{17}.$$

Substituting this back into the first equation, we have

$$8 - 6t = -\left(-\frac{88}{17}\right) \quad \Longrightarrow \quad 6t = \frac{48}{17} \quad \Longrightarrow \quad t = \frac{8}{17}.$$

But we must ensure that this is consistent with the third equation, where we find that

$$2 + 4t = 2 + 4\left(\frac{8}{17}\right) = \frac{66}{17}$$

while

$$-6 - 2s = -6 - 2\left(-\frac{88}{17}\right) = \frac{74}{17}.$$

Since these are not equal, there are no values of t and s which satisfy all three equations, and hence the two lines <u>do not</u> intersect.

[6] (b) We have

$$\begin{aligned} 8-6t &= -s \\ -5+2t &= 11+3s \\ 2+4t &= -6-2s. \end{aligned}$$

Multiplying the second equation by 3 and adding it to the first equation gives

$$-7 = 33+8s \quad \Longrightarrow \quad 8s=-40 \quad \Longrightarrow \quad s=-5.$$

Substituting this back into the first equation gives

$$8-6t=-(-5) \quad \Longrightarrow \quad 6t=3 \quad \Longrightarrow \quad t=\frac{1}{2}.$$

Again, we must ensure that this solution is consistent with the third equation, where we find that

$$2+4t = 2+4\left(\frac{1}{2}\right) = 4$$

while

$$-6-2s = -6-2(-5) = 4$$

as well. Thus we have found a solution to the equation, which upon substitution into either vector equation yields the point of intersection $(5,-4,4)$.

[2] (c) This time we can simply compare the direction vectors of the two lines and observe that

$$\begin{bmatrix} -6 \\ 2 \\ 4 \end{bmatrix} = -2\begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}.$$

Since these two vectors are parallel, we can conclude that the lines are also parallel and so they <u>do not</u> intersect.

[2] 5. (a) Setting $x=-5$, $y=0$ and $z=2$ in the equations of both planes, we see that

$$x-y+3z = -5-0+3(2) = 1$$

and

$$x-2y+3z = -5-2(0)+3(2) = 1$$

as well. Thus both equations are satisfied, and hence the point $(-5,0,2)$ must lie in both planes.

[6] (b) We have already identified a point on this line, so we just need its direction vector. Since this vector must lie in both planes, it must be perpendicular to both of their respective normals. Such a vector is given by the cross product of the two normals:

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{vmatrix} -1 & 3 \\ -2 & 3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix}\mathbf{k} = 3\mathbf{i} - 0\mathbf{j} - \mathbf{k} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}.$$

Hence the vector equation of the line of intersection is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 2 \end{bmatrix} + t \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}.$$

[4] 6. From the equation of the line, we can see that $x = 3 + t$, $y = 1 + 4t$ and $z = 6 - 3t$ for any point which lies on it. If such a point also lies in the plane then we must have

$$\begin{aligned} 5(3+t) - 2(1+4t) - (6-3t) &= 3 \\ 15 + 5t - 2 - 8t - 6 + 3t &= 3 \\ 7 &= 3, \end{aligned}$$

which is not possible. Thus the line and the plane <u>do not</u> intersect.