MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SOLUTIONS

1. (a) Observe that
$$\|\mathbf{v}\| = \sqrt{(-1)^2 + 4^2 + 3^2} = \sqrt{26}$$
so a unit vector in the direction of $\mathbf{v}$ is
$$\tilde{\mathbf{v}} = \frac{1}{\sqrt{26}}\mathbf{v} = \frac{1}{\sqrt{26}}\begin{bmatrix}-1\\4\\3\end{bmatrix}.$$

(b) Since the vector $\tilde{\mathbf{v}}$ found in (a) was a unit vector in the direction of $\mathbf{v}$, a vector of length 7 in the same direction will be
$$7\tilde{\mathbf{v}} = \frac{7}{\sqrt{26}}\mathbf{v} = \frac{7}{\sqrt{26}}\begin{bmatrix}-1\\4\\3\end{bmatrix} = \frac{1}{\sqrt{26}}\begin{bmatrix}-7\\28\\21\end{bmatrix}.$$

(c) Since the vector $\tilde{\mathbf{v}}$ found in (a) was a unit vector in the direction of $\mathbf{v}$, a vector of length 4 in the opposite direction will be
$$-4\tilde{\mathbf{v}} = -\frac{4}{\sqrt{26}}\mathbf{v} = -\frac{4}{\sqrt{26}}\begin{bmatrix}-1\\4\\3\end{bmatrix} = \frac{1}{\sqrt{26}}\begin{bmatrix}4\\-16\\-12\end{bmatrix}.$$

2. Observe that $\mathbf{u}\cdot\mathbf{v} = -3$, $\|\mathbf{u}\| = \sqrt{6}$ and $\|\mathbf{v}\| = \sqrt{2}$. So then
$$\mathbf{u}\cdot\mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta \quad\Longrightarrow\quad \cos\theta = \frac{-3}{\sqrt{6}\sqrt{2}} = -\frac{\sqrt{3}}{2}$$
and thus $\theta = \frac{5\pi}{6}$.

3. (a) We expand and simplify:
$$\begin{aligned}(\mathbf{u}+5\mathbf{w})\cdot(3\mathbf{v}-2\mathbf{u}) &= 3\mathbf{u}\cdot\mathbf{v} + 15\mathbf{w}\cdot\mathbf{v} - 2\mathbf{u}\cdot\mathbf{u} - 10\mathbf{w}\cdot\mathbf{u}\\ &= 3\mathbf{u}\cdot\mathbf{v} + 15\mathbf{v}\cdot\mathbf{w} - 2\|\mathbf{u}\|^2 - 10\mathbf{u}\cdot\mathbf{w}\\ &= 3(-3) + 15(1) - 2(4) - 10(4)\\ &= -42.\end{aligned}$$

(b) We have that
$$\begin{aligned}\|\mathbf{v}-\mathbf{w}\|^2 &= (\mathbf{v}-\mathbf{w})\cdot(\mathbf{v}-\mathbf{w})\\ &= \mathbf{v}\cdot\mathbf{v} - \mathbf{v}\cdot\mathbf{w} - \mathbf{w}\cdot\mathbf{v} + \mathbf{w}\cdot\mathbf{w}\\ &= \|\mathbf{v}\|^2 - 2\mathbf{v}\cdot\mathbf{w} + \|\mathbf{w}\|^2\\ &= 36 - 2(1) + 64\\ &= 98.\end{aligned}$$