MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 6 | **MATH 2050** | WINTER 2018

Due: Monday, March 12th, 2018. SHOW ALL WORK.

> **Note:** You should complete the worksheets for Sections 2.4 and 2.5 before you work on this assignment.

1. Consider the system

$$\left.\begin{array}{rcrcrcl} 2x & + & 3y & - & 4z & = & 7 \\ 4x & + & y & + & 9z & = & 5 \\ & & 5y & - & 17z & = & 9. \end{array}\right\}$$

   (a) Solve the corresponding homogeneous system of equations using Gaussian elimination and back-substitution.

   (b) Show that the solution of the given system can be written in the form $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_h$, where $\mathbf{x}_p$ is a particular solution of the given system and $\mathbf{x}_h$ is a solution of the corresponding homogeneous system.

2. Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 4 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 5 \\ 1 \\ 5 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ -4 \\ -3 \\ 3 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 1 \end{bmatrix}.$$

   (a) Use Gaussian elimination and back-substitution to determine whether these vectors are linearly independent or linearly dependent.

   (b) Consider the matrix A whose columns are $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$. Explain how your answer to part (a) can be used to determine whether A is invertible.

3. For each of the following matrices, use Gaussian elimination to determine the inverse of the matrix or to show that the matrix is not invertible.

   (a) $A = \begin{bmatrix} 1 & 2 & 2 & 1 \\ -2 & -4 & -4 & 0 \\ 0 & 0 & -1 & 2 \\ 3 & 3 & 3 & 1 \end{bmatrix}$

   (b) $B = \begin{bmatrix} 1 & 2 & 2 & 0 \\ -2 & -4 & -4 & 0 \\ 0 & 0 & -1 & 2 \\ 3 & 3 & 3 & 1 \end{bmatrix}$

4. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -2 & -4 & -5 \\ 2 & 1 & 0 \end{bmatrix}.$$

Find A^{-1} and use it to solve the system of equations

$$\left.\begin{array}{rcrcrcl} x & & & - & z & = & -2 \\ -2x & - & 4y & - & 5z & = & 7 \\ 2x & + & y & & & = & -4. \end{array}\right\}$$

5. Express $A = \begin{bmatrix} 2 & 2 & 0 \\ 3 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ as a product of elementary matrices.