MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 4 **MATH 2050** WINTER 2018

Due: Monday, February 26th, 2018. SHOW ALL WORK.

> **Note:** You should complete the worksheets for Sections 2.1 and 2.2 before you work on this assignment.

1. Suppose $A = \begin{bmatrix} 3 & 0 & -2 \\ 5 & -5 & 1 \\ 0 & -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ -2 & 0 \\ 7 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} -4 & -1 & 4 \\ -6 & 13 & 2 \\ 0 & 4 & -5 \end{bmatrix}$.

   (a) Compute the products AB, BA, B^TA, A^2, B^2 and B^TB, if possible. If a product does not exist, explain why not.

   (b) Solve the equation $\frac{1}{4}X - 2A = C^T$.

2. In general, matrix multiplication is not commutative; that is, given matrices A and B, $AB \neq BA$. However, prove that if A commutes with $A + B$ then A must commute with B.

3. Suppose $A = \begin{bmatrix} 3 & -1 \\ 9 & 6 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

   (a) Write the equation $A\mathbf{x} = \mathbf{b}$ as a system of linear equations.

   (b) Find A^{-1} and use it to solve the equation.

   (c) Use your result from part (b) to write $\mathbf{b}$ as a linear combination of the columns of A.

   (d) Explain why we would not be able to use the method of part (b) to solve the equation $Z\mathbf{x} = \mathbf{b}$ where $Z = \begin{bmatrix} 3 & -2 \\ -9 & 6 \end{bmatrix}$. Use another method to write $\mathbf{b}$ as a linear combination of the columns of Z.

4. Solve the matrix equation $AX + 4B = C$ for the 2×3 matrix X, given

$$A = \begin{bmatrix} 4 & 1 \\ -3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 1 & 7 \\ 0 & 1 & -5 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 4 & 3 \\ -9 & 7 & 1 \end{bmatrix}.$$

5. Suppose A and B are invertible matrices such that

$$BA^{-1}X^TB = BA^T.$$

   Find an expression for X in terms of A and B.