MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 6	**MATHEMATICS 2050**	Winter 2026

SOLUTIONS

[3] 1. (a) Using Gaussian elimination, we have

$$\begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & -9 \\ -3 & 7 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 - (-3)R_1} \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & -9 \\ 0 & 1 & -9 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & -9 \\ 0 & 0 & 0 \end{bmatrix}.$$

Since column three is not a pivot column, we set

$$\begin{aligned} z &= t \\ y &= 9z = 9t \\ x &= 3z + 2y = 3t + 18t = 21t. \end{aligned}$$

Thus the solution is $\mathbf{x}_h = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21t \\ 9t \\ t \end{bmatrix} = t \begin{bmatrix} 21 \\ 9 \\ 1 \end{bmatrix}$.

[2] (b) The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & 2 \\ 0 & 1 & -9 & 5 \\ -3 & 7 & 0 & -1 \end{array}\right] \xrightarrow{R_3 \to R_3 - (-3)R_1} \left[\begin{array}{ccc|c} 1 & -2 & -3 & 2 \\ 0 & 1 & -9 & 5 \\ 0 & 1 & -9 & 5 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & -2 & -3 & 2 \\ 0 & 1 & -9 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Since column three is not a pivot column, we set

$$\begin{aligned} z &= t \\ y &= 5 + 9z = 5 + 9t \\ x &= 2 + 3z + 2y = 2 + 3t + 10 + 18t = 12 + 21t. \end{aligned}$$

Thus the solution is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 + 21t \\ 5 + 9t \\ t \end{bmatrix} = \begin{bmatrix} 12 \\ 5 \\ 0 \end{bmatrix} + t \begin{bmatrix} 21 \\ 9 \\ 1 \end{bmatrix} = \mathbf{x}_p + \mathbf{x}_h.$$

[4] 2. We construct the matrix

$$\begin{bmatrix} 7 & 3 & 0 & 0 \\ -2 & 1 & 2 & -3 \\ 0 & -1 & 8 & -7 \\ 5 & -4 & 3 & 0 \end{bmatrix} \xrightarrow{R_1 \to \frac{1}{7}R_1} \begin{bmatrix} 1 & \frac{3}{7} & 0 & 0 \\ -2 & 1 & 2 & -3 \\ 0 & -1 & 8 & -7 \\ 5 & -4 & 3 & 0 \end{bmatrix}$$

$$\xrightarrow[R_4 \to R_4 - 5R_1]{R_2 \to R_2 - (-2)R_1} \begin{bmatrix} 1 & \frac{3}{7} & 0 & 0 \\ 0 & \frac{13}{7} & 2 & -3 \\ 0 & -1 & 8 & -7 \\ 0 & -\frac{43}{7} & 3 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \to \frac{7}{13}R_2} \begin{bmatrix} 1 & \frac{3}{7} & 0 & 0 \\ 0 & 1 & \frac{14}{13} & -\frac{21}{13} \\ 0 & -1 & 8 & -7 \\ 0 & -\frac{43}{7} & 3 & 0 \end{bmatrix}$$

$$\xrightarrow[R_4 - \left(-\frac{43}{7}\right)R_2]{R_3 \to R_3 - (-1)R_2} \begin{bmatrix} 1 & \frac{3}{7} & 0 & 0 \\ 0 & 1 & \frac{14}{13} & -\frac{21}{13} \\ 0 & 0 & \frac{118}{13} & -\frac{112}{13} \\ 0 & 0 & \frac{125}{13} & -\frac{129}{13} \end{bmatrix}$$

$$\xrightarrow{R_3 \to \frac{13}{118}R_3} \begin{bmatrix} 1 & \frac{3}{7} & 0 & 0 \\ 0 & 1 & \frac{14}{13} & -\frac{21}{13} \\ 0 & 0 & 1 & -\frac{56}{59} \\ 0 & 0 & \frac{125}{13} & -\frac{129}{13} \end{bmatrix}$$

$$\xrightarrow{R_4 \to R_4 - \left(\frac{125}{13}\right)R_3} \begin{bmatrix} 1 & \frac{3}{7} & 0 & 0 \\ 0 & 1 & \frac{14}{13} & -\frac{21}{13} \\ 0 & 0 & 1 & -\frac{56}{59} \\ 0 & 0 & 0 & -\frac{47}{59} \end{bmatrix}.$$

Hence every column is a pivot column, and so these four vectors are linearly independent.

[4] 3. (a) We have

$$\left[\begin{array}{cccc|cccc} 0 & 4 & -1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & -2 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 3 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & -2 & 0 & 1 & 0 & 0 \\ 0 & 4 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & -8 & 0 & 1 & -2 & 0 \\ 0 & 4 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_4} \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 4 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -8 & 0 & 1 & -2 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{2}R_2} \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 4 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -8 & 0 & 1 & -2 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 4R_2} \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -3 & 3 & 1 & 0 & 0 & -2 \\ 0 & 0 & 2 & -8 & 0 & 1 & -2 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to -\frac{1}{3}R_3} \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -1 & -\frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 2 & -8 & 0 & 1 & -2 & 0 \end{array}\right]$$

$$\xrightarrow[R_4 \to R_4 - 2R_3]{\substack{R_1 \to R_1 - (-1)R_3 \\ R_2 \to R_2 - \frac{1}{2}R_3}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & -\frac{1}{3} & 0 & 1 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 1 & -1 & -\frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & -6 & \frac{2}{3} & 1 & -2 & -\frac{4}{3} \end{array}\right]$$

$$\xrightarrow{R_4 \to -\frac{1}{6}R_4} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & -\frac{1}{3} & 0 & 1 & \frac{2}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 1 & -1 & -\frac{1}{3} & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 1 & -\frac{1}{9} & -\frac{1}{6} & \frac{1}{3} & \frac{2}{9} \end{array}\right]$$

$$\xrightarrow{\substack{R_1 \to R_1 - 2R_4 \\ R_3 \to R_3 - (-1)R_4}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -\frac{1}{9} & \frac{1}{3} & \frac{1}{3} & \frac{2}{9} \\ 0 & 1 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 1 & 0 & -\frac{4}{9} & -\frac{1}{6} & \frac{1}{3} & \frac{8}{9} \\ 0 & 0 & 0 & 1 & -\frac{1}{9} & -\frac{1}{6} & \frac{1}{3} & \frac{2}{9} \end{array}\right].$$

Thus

$$A^{-1} = \begin{bmatrix} -\frac{1}{9} & \frac{1}{3} & \frac{1}{3} & \frac{2}{9} \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} \\ -\frac{4}{9} & -\frac{1}{6} & \frac{1}{3} & \frac{8}{9} \\ -\frac{1}{9} & -\frac{1}{6} & \frac{1}{3} & \frac{2}{9} \end{bmatrix}.$$

[3] (b) We have

$$\left[\begin{array}{cccc|cccc} 2 & 0 & -2 & 3 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ -4 & 3 & -3 & 0 & 0 & 0 & 1 & 0 \\ -3 & -2 & 0 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & -2 & 3 & 1 & 0 & 0 & 0 \\ -4 & 3 & -3 & 0 & 0 & 0 & 1 & 0 \\ -3 & -2 & 0 & 3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - (-4)R_1 \\ R_4 \to R_4 - (-3)R_1}]{} \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & -4 & 3 & 1 & -2 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 4 & 1 & 0 \\ 0 & -5 & 3 & 3 & 0 & 3 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{2}R_2} \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & \frac{3}{2} & \frac{1}{2} & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 4 & 1 & 0 \\ 0 & -5 & 3 & 3 & 0 & 3 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[\substack{R_1 \to R_1 - (-1)R_2 \\ R_3 \to R_3 - (-1)R_2 \\ R_4 \to R_4 - (-5)R_2}]{} \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & \frac{3}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & -2 & \frac{3}{2} & \frac{1}{2} & -1 & 0 & 0 \\ 0 & 0 & -1 & \frac{3}{2} & \frac{1}{2} & 3 & 1 & 0 \\ 0 & 0 & -7 & \frac{21}{2} & \frac{5}{2} & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \to -R_3} \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & \frac{3}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & -2 & \frac{3}{2} & \frac{1}{2} & -1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & -\frac{1}{2} & -3 & -1 & 0 \\ 0 & 0 & -7 & \frac{21}{2} & \frac{5}{2} & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[\substack{R_1 \to R_1 - (-1)R_3 \\ R_2 \to R_3 - (-2)R_3 \\ R_4 \to R_4 - (-7)R_3}]{} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & -3 & -1 & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & -\frac{1}{2} & -7 & -2 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & -\frac{1}{2} & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -23 & -7 & 1 \end{array}\right].$$

Because we have a row of zeroes, it is impossible to reduce B to I. Hence B is not invertible.

[4] 4. First we bring A to reduced row-echelon form:

$$\begin{bmatrix} 0 & -3 & 6 \\ 1 & 0 & 4 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 \to -\frac{1}{3}R_2} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[\substack{R_1 \to R_1 - 4R_3 \\ R_2 \to R_2 - (-2)R_3}]{} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The elementary matrices required to do this are

$$E_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

The associated inverses are

$$E_1^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_3^{-1} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_4^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

and so

$$A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}.$$