MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 **MATHEMATICS 2050** WINTER 2026

SOLUTIONS

[4] 1. (a) The augmented matrix is

$$\left[\begin{array}{ccc|c} 0 & 3 & 2 & 9 \\ 1 & 1 & -1 & 0 \\ 2 & 5 & 0 & -7 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 3 & 2 & 9 \\ 2 & 5 & 0 & -7 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 3 & 2 & 9 \\ 0 & 3 & 2 & -7 \end{array}\right] \xrightarrow{R_2 \to \frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & \frac{2}{3} & 3 \\ 0 & 3 & 2 & -7 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 3R_2} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & \frac{2}{3} & 3 \\ 0 & 0 & 0 & -16 \end{array}\right]$$

so since the final line implies that $0 = -16$, there is no solution to the system and hence it is inconsistent.

[4] (b) The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -4 & -1 & -13 \\ -2 & 3 & 3 & 2 \\ 3 & 0 & -5 & 17 \\ 9 & 2 & 0 & -1 \end{array}\right] \xrightarrow[\substack{R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 9R_1}]{R_2 \to R_2 - (-2)R_1} \left[\begin{array}{ccc|c} 1 & -4 & -1 & -13 \\ 0 & -5 & 1 & -24 \\ 0 & 12 & -2 & 56 \\ 0 & 38 & 9 & 116 \end{array}\right]$$

$$\xrightarrow{R_2 \to -\frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & -4 & -1 & -13 \\ 0 & 1 & -\frac{1}{5} & \frac{24}{5} \\ 0 & 12 & -2 & 56 \\ 0 & 38 & 9 & 116 \end{array}\right] \xrightarrow[R_4 \to R_4 - 38R_2]{R_3 \to R_3 - 12R_2} \left[\begin{array}{ccc|c} 1 & -4 & -1 & -13 \\ 0 & 1 & -\frac{1}{5} & \frac{24}{5} \\ 0 & 0 & \frac{2}{5} & -\frac{8}{5} \\ 0 & 0 & \frac{83}{5} & -\frac{332}{5} \end{array}\right]$$

$$\xrightarrow{R_3 \to \frac{5}{2}R_3} \left[\begin{array}{ccc|c} 1 & -4 & -1 & -13 \\ 0 & 1 & -\frac{1}{5} & \frac{24}{5} \\ 0 & 0 & 1 & -4 \\ 0 & 0 & \frac{83}{5} & -\frac{332}{5} \end{array}\right] \xrightarrow{R_4 \to -\frac{83}{5}R_3} \left[\begin{array}{ccc|c} 1 & -4 & -1 & -13 \\ 0 & 1 & -\frac{1}{5} & \frac{24}{5} \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

so then we see that

$$\begin{aligned} z &= -4 \\ y &= \frac{24}{5} + \frac{1}{5}z = \frac{24}{5} + \frac{1}{5}(-4) = 4 \\ x &= -13 + z + 4y = -13 - 4 + 4(4) = -1. \end{aligned}$$

Thus the solution is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -4 \end{bmatrix}$.

[4] (c) The augmented matrix is

$$\left[\begin{array}{cccc|c} 1 & -1 & 6 & -3 & 0 \\ 1 & -9 & 8 & 1 & -2 \\ 0 & 4 & -1 & -2 & 1 \\ 1 & 11 & 3 & -9 & 3 \end{array}\right] \xrightarrow[R_4 \to R_4 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{cccc|c} 1 & -1 & 6 & -3 & 0 \\ 0 & -8 & 2 & 4 & -2 \\ 0 & 4 & -1 & -2 & 1 \\ 0 & 12 & -3 & -6 & 3 \end{array}\right]$$

$$\xrightarrow{R_2 \to -\frac{1}{8}R_2} \left[\begin{array}{cccc|c} 1 & -1 & 6 & -3 & 0 \\ 0 & 1 & -\frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 4 & -1 & -2 & 1 \\ 0 & 12 & -3 & -6 & 3 \end{array}\right] \xrightarrow[R_4 \to R_4 - 12R_2]{R_3 \to R_3 - 4R_2} \left[\begin{array}{cccc|c} 1 & -1 & 6 & -3 & 0 \\ 0 & 1 & -\frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

The third and fourth columns are non-pivot columns, so we let $z = t$ and $y = s$. Then

$$x = \frac{1}{4} + \frac{1}{2}z + \frac{1}{4}y = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}s.$$

Finally,

$$w = 3z - 6y + x = 3t - 6s + \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}s = \frac{1}{4} + \frac{7}{2}t - \frac{23}{4}s.$$

So the solution is

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{7}{2}t - \frac{23}{4}s \\ \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}s \\ s \\ t \end{bmatrix} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} \frac{7}{2} \\ \frac{1}{2} \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} -\frac{23}{4} \\ \frac{1}{4} \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 7 \\ 1 \\ 0 \\ 2 \end{bmatrix} + s \begin{bmatrix} -23 \\ 1 \\ 4 \\ 0 \end{bmatrix}.$$

This system has an infinite number of solutions.

[8] 2. First we row-reduce the augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & 8 \\ -1 & -1 & 1 & -4 \\ 2 & 0 & 1 & 4 \\ a & b & c & 0 \end{array}\right] \xrightarrow[\substack{R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 - aR_1}]{R_2 \to R_2 - (-1)R_1} \left[\begin{array}{ccc|c} 1 & 3 & -4 & 8 \\ 0 & 2 & -3 & 4 \\ 0 & -6 & 9 & -12 \\ 0 & b-3a & c+4a & -8a \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 3 & -4 & 8 \\ 0 & 1 & -\frac{3}{2} & 2 \\ 0 & -6 & 9 & -12 \\ 0 & b-3a & c+4a & -8a \end{array}\right] \xrightarrow[R_4 \to R_4 - (b-3a)R_2]{R_3 \to R_3 - (-6)R_2} \left[\begin{array}{ccc|c} 1 & 3 & -4 & 8 \\ 0 & 1 & -\frac{3}{2} & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & c + \frac{3}{2}b - \frac{1}{2}a & -2b - 2a \end{array}\right]$$

$$\xrightarrow{R_3 \leftrightarrow R_4} \left[\begin{array}{ccc|c} 1 & 3 & -4 & 8 \\ 0 & 1 & -\frac{3}{2} & 2 \\ 0 & 0 & c + \frac{3}{2}b - \frac{1}{2}a & -2b - 2a \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Note that we've chosen not to write the final pivot as a 1 because we cannot be certain that $c + \frac{3}{2}b - \frac{1}{2}a \neq 0$.

(a) To obtain a unique solution, we must ensure that all of the columns are pivot columns, so we need

$$c + \frac{3}{2}b - \frac{1}{2}a \neq 0 \quad \text{or} \quad c \neq \frac{1}{2}a - \frac{3}{2}b.$$

(b) To obtain an infinite number of solutions, we must have at least one non-pivot column. This can only happen if

$$c + \frac{3}{2}b - \frac{1}{2}a = 0 \quad \text{or} \quad c = \frac{1}{2}a - \frac{3}{2}b.$$

We also need

$$-2b - 2a = 0 \quad \text{or} \quad a = -b.$$

We can combine these conditions to obtain the single condition

$$c = -2b = 2a.$$

(c) To obtain an inconsistent system, we must have a row of zeroes on the lefthand side with a non-zero entry on the righthand side. Thus we must have

$$c + \frac{3}{2}b - \frac{1}{2}a = 0 \quad \text{or} \quad c = \frac{1}{2}a - \frac{3}{2}b$$

and

$$-2b - 2a \neq 0 \quad \text{or} \quad a \neq -b.$$