MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.6 **Math 2050 Worksheet** WINTER 2026

SOLUTIONS

1. We first row-reduce the matrix of coefficients A to row-echelon form using only the third elementary row operation:

$$A = \begin{bmatrix} 5 & 2 & -1 \\ 1 & 0 & 4 \\ -2 & 0 & 1 \end{bmatrix} \xrightarrow[R_3 \to R_3 - \left(-\frac{2}{5}\right) R_1]{R_2 \to R_2 - \frac{1}{5} R_1} \begin{bmatrix} 5 & 2 & -1 \\ 0 & -\frac{2}{5} & \frac{21}{5} \\ 0 & \frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - (-2) R_2} \begin{bmatrix} 5 & 2 & -1 \\ 0 & -\frac{2}{5} & \frac{21}{5} \\ 0 & 0 & 9 \end{bmatrix} = U.$$

Now we compose the matrix L. We subtracted $\frac{1}{5}$ times the 1st row from the 2nd row, so the $(2,1)$ element is $\frac{1}{5}$. We subtracted $-\frac{2}{5}$ times the 1st row from the 3rd row, so the $(3,1)$ element is $-\frac{2}{5}$. We subtracted -2 times the 2nd row from the 3rd row, so the $(3,2)$ element is -2. Hence

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & 1 & 0 \\ -\frac{2}{5} & -2 & 1 \end{bmatrix}.$$

So now we want to solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 12 \\ -13 \\ -1 \end{bmatrix}$. This is equivalent to $LU\mathbf{x} = \mathbf{b}$, and so first we solve $L\mathbf{y} = \mathbf{b}$ by forward-substitution. We have

$$y_1 = 12$$

$$y_2 = -13 - \frac{1}{5} y_1 = -\frac{77}{5},$$

$$y_3 = -1 + 2y_2 + \frac{2}{5} y_1 = -1 - \frac{154}{5} + \frac{24}{5} = -27.$$

Now we use back-substitution to solve $U\mathbf{x} = \mathbf{y}$ and so

$$x_3 = \frac{1}{9}(-27) = -3$$

$$x_2 = -\frac{5}{2}\left(-\frac{77}{5} - \frac{21}{5} x_3\right) = 7$$

$$x_1 = \frac{1}{5}(12 + x_3 - 2x_2) = -1.$$