MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 1.3 — **Math 2050 Worksheet** — WINTER 2026

SOLUTIONS

1. (a) First note that

$$2\mathbf{u} - k\mathbf{v} = 2\begin{bmatrix}4\\-1\\1\end{bmatrix} - k\begin{bmatrix}1\\0\\-3\end{bmatrix} = \begin{bmatrix}8-k\\-2\\2+3k\end{bmatrix}.$$

We want $\mathbf{u}\cdot(2\mathbf{u}-k\mathbf{v}) = 0$, so we set

$$0 = \begin{bmatrix}4\\-1\\1\end{bmatrix}\cdot\begin{bmatrix}8-k\\-2\\2+3k\end{bmatrix} = 4(8-k)+(-1)(-2)+1(2+3k) = 36-k,$$

and hence $k = 36.$

(b) We want $\mathbf{v}\cdot(2\mathbf{u}-k\mathbf{v}) = 0$, so we set

$$0 = \begin{bmatrix}1\\0\\-3\end{bmatrix}\cdot\begin{bmatrix}8-k\\-2\\2+3k\end{bmatrix} = 1(8-k)+0(-2)+(-3)(2+3k) = 2-10k,$$

so $10k = 2$ and $k = \frac{1}{5}.$

(c) We have

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix}-1 & 1\\0 & -3\end{vmatrix}\mathbf{i} - \begin{vmatrix}4 & 1\\1 & -3\end{vmatrix}\mathbf{j} + \begin{vmatrix}4 & -1\\1 & 0\end{vmatrix}\mathbf{k} = \begin{bmatrix}3\\13\\1\end{bmatrix}.$$

But notice that

$$\begin{bmatrix}3\\13\\1\end{bmatrix}\cdot\begin{bmatrix}8-k\\-2\\2+3k\end{bmatrix} = 3(8-k)+13(-2)+1(2+3k) = 0$$

identically — that is, for any value of k.

Alternatively, we can simply observe that $\mathbf{u}\times\mathbf{v}$ is orthogonal to both u and v, and hence to any linear combination of u and v, implying that *any* scalar k will work.

Or, we could notice that

$$(\mathbf{u}\times\mathbf{v})\cdot(2\mathbf{u}-k\mathbf{v}) = 2(\mathbf{u}\times\mathbf{v})\cdot\mathbf{u} - k(\mathbf{u}\times\mathbf{v})\cdot\mathbf{v} = 0-0 = 0$$

for any k.

2. Note that we want

$$(k\mathbf{u}+\mathbf{v})\cdot\mathbf{v} = k\mathbf{u}\cdot\mathbf{v}+\mathbf{v}\cdot\mathbf{v} = 0 \quad\Longrightarrow\quad k = -\frac{\mathbf{v}\cdot\mathbf{v}}{\mathbf{u}\cdot\mathbf{v}} = -\frac{\|\mathbf{v}\|^2}{\mathbf{u}\cdot\mathbf{v}},$$

where we know that $\mathbf{u}\cdot\mathbf{v}\neq 0$ since $\mathbf{u}$ and $\mathbf{v}$ are not orthogonal.

3. First we compute the normal to the plane:

$$\mathbf{n} = \mathbf{u}\times\mathbf{v} = \begin{vmatrix}4 & 5\\ 3 & 0\end{vmatrix}\mathbf{i} - \begin{vmatrix}2 & 5\\ -1 & 0\end{vmatrix}\mathbf{j} + \begin{vmatrix}2 & 4\\ -1 & 3\end{vmatrix} = -15\mathbf{i} - 5\mathbf{j} + 10\mathbf{k} = \begin{bmatrix}-15\\ -5\\ 10\end{bmatrix}.$$

Hence the equation of the plane is of the form

$$-15x - 5y + 10z = d,$$

and since the origin $(0,0,0)$ is in the plane, we have that $d = 0$. So the equation of the plane is

$$-15x - 5y + 10z = 0.$$

4. First we compute two vectors in the plane, say, $\overrightarrow{AB} = \begin{bmatrix}6\\ -2\\ 3\end{bmatrix}$ and $\overrightarrow{AC} = \begin{bmatrix}1\\ -5\\ -2\end{bmatrix}$. A normal to these two vectors is

$$\mathbf{n} = \overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix}-2 & 3\\ -5 & -2\end{vmatrix}\mathbf{i} - \begin{vmatrix}6 & 3\\ 1 & -2\end{vmatrix}\mathbf{j} + \begin{vmatrix}6 & -2\\ 1 & -5\end{vmatrix}\mathbf{k} = 19\mathbf{i} + 15\mathbf{j} - 28\mathbf{k}.$$

The plane has an equation of the form

$$19x + 15y - 28z = d.$$

Using the point $(0,4,-1)$, we see that $d = 19(0) + 15(4) - 28(-1) = 88$. So the equation of the plane is

$$19x + 15y - 28z = 88.$$

5. A direction vector for this line is

$$\mathbf{d} = \overrightarrow{PQ} = \begin{bmatrix}4\\ -9\\ 0\end{bmatrix}.$$

So the vector equation of the line is of the form

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ 6\\ 0\end{bmatrix} + t\begin{bmatrix}4\\ -9\\ 0\end{bmatrix}.$$

The parametric equations are

$$\begin{cases} x = 1 + 4t \\ y = 6 - 9t \\ z = 0. \end{cases}$$

6. Using s as the parameter for the second equation, we want to show that, by replacing t by some expression involving s in the first equation, we can set $x = 1 - 3t = 16 + 6s$ (and similarly for the equations for y and z). This means that

$$3t = -15 - 6s \quad \Longrightarrow \quad t = -5 - 2s.$$

We must verify that this works for the other two equations. For y, we have

$$y = 4 + 3t = 4 + 3(-5 - 2s) = -11 - 6s$$

as desired, and for z, we get

$$z = 9 + 2t = 9 + 2(-5 - 2s) = -1 - 4s,$$

also as desired. Thus replacing t in the first equation with $-5 - 2s$ gives rise to the second equation, so the two equations are equivalent.

Alternatively, note that if we look for the points of intersection of the two lines, we obtain the system of equations

$$\begin{aligned} 1 - 3t &= 16 + 6s \\ 4 + 3t &= -11 - 6s \\ 9 + 2t &= -1 - 4s. \end{aligned}$$

No matter how we try to combine these equations, we always find that s and t cancel out, and we wind up with the expression $0 = 0$, which of course is always true. This means that there is an infinite number of solutions, and therefore an infinite number of points of intersection. The only way this can happen is if the two lines are actually the same line.

7. Using s as the parameter for the second equation, we set the equations of the two lines equal to each other:

$$\begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} + s \begin{bmatrix} 6 \\ 3 \\ -12 \end{bmatrix}$$

yielding the system of equations

$$\begin{aligned} 4 + 2t &= 2 + 6s \\ t &= -3 + 3s \\ -4t &= 4 - 12s. \end{aligned}$$

Substituting the second equation in the third equation gives

$$-4(-3+3s) = 4 - 12s \implies 12 - 12s = 4 - 12s \implies 8 = 0,$$

which is impossible. Hence the system has no solution, and so there is no point of intersection.

(In fact, we can see that if we let $\mathbf{d_1}$ be the direction vector of the first line and $\mathbf{d_2}$ be the direction vector of the second line, then $\mathbf{d_2} = 3\mathbf{d_1}$ so the two lines are parallel. So either they're the same line — which they're not, because you can show that $(4, 0, 0)$ doesn't lie on the second line, for instance — or they're parallel so they never intersect.)

8. The direction vector of the line is $\mathbf{d} = \begin{bmatrix} -8 \\ 3 \\ 3 \end{bmatrix}$ and the normal vector to the plane is $\mathbf{n} = \begin{bmatrix} 1 \\ -6 \\ 5 \end{bmatrix}$.

Observe that

$$\mathbf{d} \cdot \mathbf{n} = -8 - 18 + 15 = -11,$$

so these vectors are not orthogonal, and therefore the line and the plane are not parallel. Thus they must intersect.

To find the point of intersection, we see from the equation for the line that

$$\begin{aligned} x &= -8t \\ y &= 3 + 3t \\ z &= -1 + 3t. \end{aligned}$$

Substituting these into the equation of the plane gives

$$-8t - 6(3+3t) + 5(-1+3t) = 10 \implies -11t = 33$$

and so $t = -3$. Substituting this value of t into the equation of the line, we see that the line and the plane intersect at the point $(24, -6, -10)$.