9. a) $A-\lambda I = \begin{bmatrix} -2-\lambda & 1 \\ 4 & 1-\lambda \end{bmatrix}$

The characteristic polynomial is

$$\det(A-\lambda I) = (-2-\lambda)(1-\lambda) - 4$$
$$= -2 + 2\lambda - \lambda + \lambda^2 - 4$$
$$= \lambda^2 + \lambda - 6$$

We set $\lambda^2+\lambda-6=0$

$$(\lambda+3)(\lambda-2)=0$$
$$\lambda=-3,\ \lambda=2$$

b) $\underline{\lambda=-3}$

$A-\lambda I = A+3I = \begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 4R_1} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad x_2 = t$, $x_1 = -x_2 = -t$

$\underline{x} = t\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ so an eigenvector is $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\underline{\lambda=2}$

$A-\lambda I = A-2I = \begin{bmatrix} -4 & 1 \\ 4 & -1 \end{bmatrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{bmatrix} -4 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad x_2 = t$, $x_1 = \frac{1}{4}x_2 = \frac{1}{4}t$

$\underline{x} = t\begin{bmatrix} 1/4 \\ 1 \end{bmatrix} = t\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ so an eigenvector is $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

c) $P = \begin{bmatrix} -1 & 1 \\ 1 & 4 \end{bmatrix}$

$D = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$