8. a) $\det(A) = -2\begin{vmatrix} 2 & -2 & 1 \\ -1 & 2 & 0 \\ -4 & 3 & -2 \end{vmatrix} + 0 - 0 + 4\begin{vmatrix} 1 & 2 & -2 \\ 3 & -1 & 2 \\ 0 & -4 & 3 \end{vmatrix}$

$$\begin{vmatrix} 2 & -2 & 1 \\ -1 & 2 & 0 \\ -4 & 3 & -2 \end{vmatrix} = 1\begin{vmatrix} -2 & 1 \\ 3 & -2 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ -4 & -2 \end{vmatrix} - 0$$

$$= 1 + 2(0) = 1$$

$$\begin{vmatrix} 1 & 2 & -2 \\ 3 & -1 & 2 \\ 0 & -4 & 3 \end{vmatrix} = 0 - (-4)\begin{vmatrix} 1 & -2 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= 4(8) + 3(-7) = 11$$

$\det(A) = -2 \cdot 1 + 4 \cdot 11 \boxed{= 42}$

b) $\det(A^2 B^T A^{-1}) = \det(A^2)\det(B^T)\det(A^{-1})$

$$= [\det(A)]^2 \det(B) \cdot \frac{1}{\det(A)}$$

$$= \det(A)\det(B)$$

$$= 2\det(B)$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \xrightarrow{R_1 \to R_1 + R_2} \begin{bmatrix} a+c & b+d \\ c & d \end{bmatrix} \xrightarrow{R_2 \to 2R_2} \begin{bmatrix} a+c & b+d \\ 2c & 2d \end{bmatrix} = B^T$$

$\det(B) = \det(B^T) = 2\det(A) = 2 \cdot 2 = 4$

Thus $\det(A^2 B^T A^{-1}) = 2 \cdot 4 \boxed{= 8}$