4. $\left[\begin{array}{ccc|c} 0 & 1 & 2k & 0 \\ 1 & 2 & 6 & 2 \\ k & 0 & 2 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 2 & 6 & 2 \\ 0 & 1 & 2k & 0 \\ k & 0 & 2 & 1 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 - kR_1} \left[\begin{array}{ccc|c} 1 & 2 & 6 & 2 \\ 0 & 1 & 2k & 0 \\ 0 & -2k & 2-6k & 1-2k \end{array}\right] \xrightarrow{R_3 \to R_3 + 2kR_2} \left[\begin{array}{ccc|c} 1 & 2 & 6 & 2 \\ 0 & 1 & 2k & 0 \\ 0 & 0 & 2-6k+4k^2 & 1-2k \end{array}\right]$

a) $2-6k+4k^2=0$

$2k^2-3k+1=0$

$(2k-1)(k-1)=0$

$k=\frac{1}{2}, k=1$

$1-2k=0$

$k=\frac{1}{2}$

There will be an infinite number of solutions if $k=\frac{1}{2}$.

b) There will be no solutions if $k=1$.

c) There will be a unique solution if $k \neq \frac{1}{2}$ and $k \neq 1$.