11. a) Let $\underline{u}$ be any vector in Π. Then $\underline{u} = k\underline{e} + l\underline{f}$ for scalars k and l.

We assume that $\underline{v} \cdot \underline{e} = 0$ and $\underline{v} \cdot \underline{f} = 0$.

We want to show that $\underline{v} \cdot \underline{u} = 0$.

We have
$$\begin{aligned}
\underline{v} \cdot \underline{u} &= \underline{v} \cdot (k\underline{e} + l\underline{f}) \\
&= k\underline{v} \cdot \underline{e} + l\underline{v} \cdot \underline{f} \\
&= k(0) + l(0) \\
&= 0
\end{aligned}$$
so $\underline{v}$ is orthogonal to every vector in Π.

b) We want to show that $AA = I$.

We have
$$\begin{aligned}
AA &= (I - 2E)(I - 2E) \\
&= I^2 - 2EI - I(2E) + (2E)(2E) \\
&= I^2 - 2EI - 2IE + 4E^2 \\
&= I - 2E - 2E + 4E^2 \\
&= I - 4E + 4E^2 \\
&= I - 4E + 4E \quad \text{because } E^2 = E \\
&= I
\end{aligned}$$
so $A^{-1} = A$

c) We are given that $A\underline{x} = \lambda\underline{x}$ where $\underline{x}$ is an eigenvector.

Then if $A^2 = I$, we have
$$\begin{aligned}
A^2\underline{x} &= I\underline{x} \\
A(A\underline{x}) &= \underline{x} \\
A(\lambda\underline{x}) &= \underline{x} \\
\lambda A\underline{x} &= \underline{x} \\
\lambda(\lambda\underline{x}) &= \underline{x}
\end{aligned}$$
$$\rightarrow \lambda^2\underline{x} = \underline{x}$$

Thus $\lambda^2 = 1$
$$\lambda = \pm 1.$$