10. $A-\lambda I=\begin{bmatrix} 2-\lambda & -1 \\ 3 & 2-\lambda \end{bmatrix}$

$\det(A-\lambda I)=(2-\lambda)^2+3$

$=4-4\lambda+\lambda^2+3$

$=\lambda^2-4\lambda+7$

We set $\lambda^2-4\lambda+7=0$

$$\lambda=\frac{4\pm\sqrt{16-28}}{2}=\frac{4\pm\sqrt{-12}}{2}$$

$$=\frac{4\pm 2\sqrt{3}i}{2}=2\pm\sqrt{3}i$$

$\lambda_1=2+\sqrt{3}i$

$\lambda_2=2-\sqrt{3}i$

$$\frac{\lambda_1}{\lambda_2}=\frac{2+\sqrt{3}i}{2-\sqrt{3}i}\cdot\frac{2+\sqrt{3}i}{2+\sqrt{3}i}=\frac{4+2\sqrt{3}i+2\sqrt{3}i+3i^2}{4-3i^2}$$

$$=\frac{1+4\sqrt{3}i}{7}=\frac{1}{7}+\frac{4\sqrt{3}}{7}i$$

$$\frac{\lambda_2}{\lambda_1}=\frac{2-\sqrt{3}i}{2+\sqrt{3}i}\cdot\frac{2-\sqrt{3}i}{2-\sqrt{3}i}=\frac{4-2\sqrt{3}i-2\sqrt{3}i+3i^2}{4-3i^2}$$

$$=\frac{1-4\sqrt{3}i}{7}=\frac{1}{7}-\frac{4\sqrt{3}}{7}i$$