

$$10. \quad A - \lambda I = \begin{bmatrix} 2-\lambda & -1 \\ 3 & 2-\lambda \end{bmatrix}$$

$$\begin{aligned} \det(A - \lambda I) &= (2-\lambda)^2 + 3 \\ &= 4 - 4\lambda + \lambda^2 + 3 \\ &= \lambda^2 - 4\lambda + 7 \end{aligned}$$

$$\text{We set } \lambda^2 - 4\lambda + 7 = 0$$

$$\begin{aligned} \lambda &= \frac{4 \pm \sqrt{16 - 28}}{2} = \frac{4 \pm \sqrt{-12}}{2} \\ &= \frac{4 \pm 2\sqrt{3}i}{2} = 2 \pm \sqrt{3}i \end{aligned}$$

$$\lambda_1 = 2 + \sqrt{3}i$$

$$\lambda_2 = 2 - \sqrt{3}i$$

$$\begin{aligned} \frac{\lambda_1}{\lambda_2} &= \frac{2 + \sqrt{3}i}{2 - \sqrt{3}i} \cdot \frac{2 + \sqrt{3}i}{2 + \sqrt{3}i} = \frac{4 + 2\sqrt{3}i + 2\sqrt{3}i + 3i^2}{4 - 3i^2} \\ &= \frac{1 + 4\sqrt{3}i}{7} = \frac{1}{7} + \frac{4\sqrt{3}}{7}i \end{aligned}$$

$$\begin{aligned} \frac{\lambda_2}{\lambda_1} &= \frac{2 - \sqrt{3}i}{2 + \sqrt{3}i} \cdot \frac{2 - \sqrt{3}i}{2 - \sqrt{3}i} = \frac{4 - 2\sqrt{3}i - 2\sqrt{3}i + 3i^2}{4 - 3i^2} \\ &= \frac{1 - 4\sqrt{3}i}{7} = \frac{1}{7} - \frac{4\sqrt{3}}{7}i \end{aligned}$$