1. a) $\|\underline{u}\| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$

A unit vector is $\frac{\underline{u}}{\|\underline{u}\|} = \frac{1}{\sqrt{6}} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \frac{\sqrt{6}}{6} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

A unit vector in the opposite direction to $\underline{u}$ is

$$-\frac{\sqrt{6}}{6} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

b) $\underline{u} \cdot \underline{v} = \|\underline{u}\| \|\underline{v}\| \cos(\theta)$

$\underline{u} \cdot \underline{v} = 2(1) + (-1)(1) + 1(2) = 3$

$\|\underline{v}\| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$

Then $3 = \sqrt{6} \cdot \sqrt{6} \cos(\theta)$

$\cos(\theta) = \frac{3}{6} = \frac{1}{2} \rightarrow \boxed{\theta = \frac{\pi}{3}}$

c) We set $(\underline{u} + k\underline{v}) \cdot (\underline{u} - k\underline{v}) = 0$

IN THE VIDEO, I MISSPOKE AND DESCRIBED THIS AS "$k\underline{u}$" BUT IT SHOULD BE "$k\underline{v}$"

$\underline{u} \cdot \underline{u} + k\underline{v} \cdot \underline{u} - \underline{u} \cdot k\underline{v} - (k\underline{v}) \cdot (k\underline{v}) = 0$

$\underline{u} \cdot \underline{u} + k(\underline{u} \cdot \underline{v}) - k(\underline{u} \cdot \underline{v}) - k^2(\underline{v} \cdot \underline{v}) = 0$

$\|\underline{u}\|^2 - k^2 \|\underline{v}\|^2 = 0$

$(\sqrt{6})^2 - k^2(\sqrt{6})^2 = 0$

$k^2 = \frac{6}{6} = 1$

$\boxed{k = \pm 1}$