

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 1

MATHEMATICS 2000

OCTOBER 15TH, 2018

Name

MUN Number

[14] 1. Find the limit of each sequence $\{a_i\}$, or explain why it diverges.

(a) $a_i = \frac{5 \cdot 2^{3i+1}}{7^i - 3^{2i-1}}$

(b) $a_i = (-1)^i \frac{(i-3)^2}{i(i^2+4)}$

(c) $a_i = \frac{\ln(e^i + 2)}{e^i + 3}$

- [6] 2. Use the Bounded Monotonic Sequence Theorem to show that $\{a_i\}$ is convergent, where

$$a_i = 3 - \frac{8i}{e^{2i}}.$$

Note that $a_1 = 3 - \frac{8}{e^2} \approx 1.92$.

- [6] 3. Sketch an example of each of the following sequences, or briefly explain why such a sequence is not possible.

- (a) A convergent sequence $\{a_i\}$ which is not monotonic.
- (b) A divergent sequence $\{b_i\}$ which is monotonic.
- (c) A divergent sequence $\{c_i\}$ which is bounded.

[4] 4. Find and sketch the domain of the function $f(x, y) = \frac{\ln(y)}{\sqrt{x^2 - y}}$.

[5] 5. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{6x^2y}{x^3 + 2y^3}$ or show that the limit does not exist.

[5] 6. Determine whether $z = \ln(x + y^2)$ is a solution of the partial differential equation

$$2x \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = y \frac{\partial^2 z}{\partial x \partial y}.$$