MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 2 **MATH 2000** FALL 2018

SOLUTIONS

[4] 1. (a) The sequence is defined by a rational expression, so we can write

$$\lim_{i\to\infty} a_i = \lim_{i\to\infty} \frac{24i^3 - 40i^2}{12i^3 - 16i^2 - 3i} \cdot \frac{\frac{1}{i^3}}{\frac{1}{i^3}} = \lim_{i\to\infty} \frac{24 - \frac{40}{i}}{12 - \frac{16}{i} - \frac{3}{i^2}} = \frac{24 - 0}{12 - 0 - 0} = 2.$$

[5] (b) This is a geometric expression, so we can write it as

$$a_i = \frac{2^{4i} \cdot 2^{-1}}{3^{3i} \cdot 3^2} = \frac{1}{18} \cdot \frac{16^i}{27^i} = \frac{1}{18}\left(\frac{16}{27}\right)^i.$$

Hence this is a geometric sequence with common ratio $|r| = \frac{16}{27} < 1$ and so

$$\lim_{i\to\infty} a_i = \lim_{i\to\infty} \frac{1}{18}\left(\frac{16}{27}\right)^i = 0.$$

[5] (c) This expression is comprised of geometric terms, so first we rewrite it as

$$a_i = \frac{3^{2i} \cdot 3^1 + 6^i}{5 - 7^i \cdot 7^2} = \frac{3 \cdot 9^i + 6^i}{5 - 49 \cdot 7^i}.$$

The dominant term in the denominator is 7^i, so we have

$$\lim_{i\to\infty} a_i = \lim_{i\to\infty} \frac{3 \cdot 9^i + 6^i}{5 - 49 \cdot 7^i} \cdot \frac{\frac{1}{7^i}}{\frac{1}{7^i}} = \lim_{i\to\infty} \frac{3\left(\frac{9}{7}\right)^i + \left(\frac{6}{7}\right)^i}{5\left(\frac{1}{7}\right)^i - 49} = \lim_{i\to\infty} \frac{3\left(\frac{9}{7}\right)^i + 0}{0 - 49} = -\frac{1}{49}\lim_{i\to\infty}\left(\frac{9}{7}\right)^i,$$

which <u>does not exist</u> because the remaining expression is a geometric sequence with common ratio $|r| = \frac{9}{7} > 1$.

[5] (d) We can write

$$\lim_{i\to\infty} a_i = \lim_{i\to\infty} \frac{(-1)^i}{3^i} \cdot \lim_{i\to\infty} \frac{i}{(i+1)^2},$$

as long as the two limits on the righthand side exist. First,

$$\lim_{i\to\infty} \frac{(-1)^i}{3^i} = \lim_{i\to\infty}\left(-\frac{1}{3}\right)^i = 0$$

since this is a geometric sequence with common ratio $|r| = \frac{1}{3} < 1$. Next,

$$\lim_{i\to\infty} \frac{i}{(i+1)^2} = \lim_{i\to\infty} \frac{i}{i^2 + 2i + 1} \cdot \frac{\frac{1}{i^2}}{\frac{1}{i^2}} = \lim_{i\to\infty} \frac{\frac{1}{i}}{1 + \frac{2}{i} + \frac{1}{i^2}} = \frac{0}{1 + 0 + 0} = 0.$$

Hence

$$\lim_{i\to\infty} a_i = 0 \cdot 0 = 0.$$

[2] 2. (a) By direct substitution,

$$\lim_{(x,y)\to(4,8)} \frac{2y-x}{x^2-2xy-3x+6y} = \frac{16-4}{16-64-12+48} = -1.$$

[5] (b) This time, direct substitution results in a $\frac{0}{0}$ indeterminate form. However, we can factor both the numerator and the denominator and write

$$\begin{aligned}\lim_{(x,y)\to(8,4)} \frac{2y-x}{x^2-2xy-3x+6y} &= \lim_{(x,y)\to(8,4)} \frac{-(x-2y)}{x(x-2y)-3(x-2y)} \\ &= \lim_{(x,y)\to(8,4)} \frac{-(x-2y)}{(x-2y)(x-3)} \\ &= \lim_{(x,y)\to(8,4)} \frac{-1}{x-3} \\ &= -\frac{1}{5}.\end{aligned}$$

[6] (c) Again, direct substitution results in a $\frac{0}{0}$ indeterminate form, and this time it is not possible to factor and cancel. Thus we will attempt to prove that the limit does not exist. Along the line $y = 1$, the limit becomes

$$\lim_{(x,y)\to(-3,1)} \frac{x+4y-1}{3y-2x-9} = \lim_{x\to-3} \frac{x+4-1}{3-2x-9} = \lim_{x\to-3} \frac{x+3}{-2(x+3)} = \lim_{x\to-3} \frac{1}{-2} = -\frac{1}{2}.$$

Along the line $x = -3$, it becomes

$$\lim_{(x,y)\to(-3,1)} \frac{x+4y-1}{3y-2x-9} = \lim_{y\to1} \frac{-3+4y-1}{3y+6-9} = \lim_{y\to1} \frac{4(y-1)}{3(y-1)} = \lim_{y\to1} \frac{4}{3} = \frac{4}{3}.$$

Since these limits are not equal, we conclude that the limit does not exist.

[8] (d) Once again, direct substitution results in a $\frac{0}{0}$ indeterminate form, and we will attempt to demonstrate that the limit does not exist. Along $y = 0$, the limit becomes

$$\lim_{(x,y)\to(0,0)} \frac{7xy^2}{x^2+y^4} = \lim_{x\to0} \frac{0}{x^2} = \lim_{x\to0} 0 = 0.$$

Along $x = 0$, we have

$$\lim_{(x,y)\to(0,0)} \frac{7xy^2}{x^2+y^4} = \lim_{y\to0} \frac{0}{y^4} = \lim_{y\to0} 0 = 0.$$

Along $y = x$ the limit is

$$\lim_{(x,y)\to(0,0)} \frac{7xy^2}{x^2+y^4} = \lim_{x\to0} \frac{7x^3}{x^2+x^4} = \lim_{x\to0} \frac{7x}{1+x^2} = 0.$$

Along $y = x^2$, we can write

$$\lim_{(x,y)\to(0,0)} \frac{7xy^2}{x^2+y^4} = \lim_{x\to0} \frac{7x^5}{x^2+x^8} = \lim_{x\to0} \frac{7x^3}{1+x^6} = 0.$$

But along $x = y^2$, we have

$$\lim_{(x,y)\to(0,0)} \frac{7xy^2}{x^2 + y^4} = \lim_{y\to 0} \frac{7y^4}{y^4 + y^4} = \lim_{y\to 0} \frac{7}{2} = \frac{7}{2}.$$

Since this is not the same as the limit along the other paths, it must be that the limit does not exist.