

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.6

Math 2000 Worksheet

FALL 2018

SOLUTIONS

1. (a) First observe that

$$\int f(x, y) dx = \int \frac{y}{x^2 + y^2} dx = y \cdot \frac{1}{y} \arctan\left(\frac{x}{y}\right) + g(y) = \arctan\left(\frac{x}{y}\right) + C(y).$$

To integrate with respect to y , we use u -substitution. We let $u = x^2 + y^2$ so $\frac{1}{2}du = y dy$ and thus

$$\int f(x, y) dy = \int \frac{y}{x^2 + y^2} dy = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| + C(x) = \frac{1}{2} \ln(x^2 + y^2) + C(x).$$

- (b) To integrate with respect to x , we use integration by parts, letting $w = \ln(x)$ so $dw = \frac{1}{x} dx$ and $dv = xy dx$ so $v = \frac{1}{2}x^2y$. Thus

$$\begin{aligned} \int f(x, y) dx &= \int xy \ln(x) dx = \frac{1}{2}x^2y \cdot \ln(x) - \int \left(\frac{1}{2}x^2y \cdot \frac{1}{x}\right) dx \\ &= \frac{1}{2}x^2y \ln(x) - \frac{1}{2} \int xy dx = \frac{1}{2}x^2y \ln(x) - \frac{1}{4}x^2y + C(y). \end{aligned}$$

Next,

$$\int f(x, y) dy = \int xy \ln(x) dy = \frac{1}{2}xy^2 \ln(x) + C(x).$$

2. (a) We have

$$\begin{aligned} \int_0^\pi \int_0^4 y \cos\left(\frac{x}{4}\right) dy dx &= \int_0^\pi \left[\frac{1}{2}y^2 \cos\left(\frac{x}{4}\right) \right]_{y=0}^{y=4} dx \\ &= \frac{1}{2} \int_0^\pi \left[16 \cos\left(\frac{x}{4}\right) \right] dx \\ &= 8 \int_0^\pi \cos\left(\frac{x}{4}\right) dx \\ &= 8 \left[4 \sin\left(\frac{x}{4}\right) \right]_0^\pi \\ &= 32 \left[\sin\left(\frac{\pi}{4}\right) - \sin(0) \right] \\ &= 32 \cdot \frac{\sqrt{2}}{2} \\ &= 16\sqrt{2}. \end{aligned}$$

(b) We have

$$\begin{aligned}\int_{-2}^1 \int_0^6 \frac{4y+3}{7-x} dx dy &= \int_{-2}^1 \left[-(4y+3) \ln|7-x| \right]_0^6 dy \\ &= \int_{-2}^1 -(4y+3)[\ln(1) - \ln(7)] dy \\ &= \ln(7) \int_{-2}^1 (4y+3) dy \\ &= \ln(7) \left[2y^2 + 3y \right]_{-2}^1 \\ &= 3 \ln(7).\end{aligned}$$

(c) We have

$$\begin{aligned}\int_1^5 \int_0^{\pi/4} 3y^2 \sec^2(x) dx dy &= \int_1^5 \left[3y^2 \tan(x) \right]_0^{\pi/4} dy \\ &= \int_1^5 3y^2 dy \\ &= \left[y^3 \right]_1^5 \\ &= 124.\end{aligned}$$

3. We can integrate in either order. If we integrate with respect to x first, then we write

$$\begin{aligned}\iint_R (x^2 - xy + y^3 - 4) dA &= \int_{-2}^1 \int_{-1}^2 (x^2 - xy + y^3 - 4) dx dy \\ &= \int_{-2}^1 \left[\frac{1}{3}x^3 - \frac{1}{2}x^2y + xy^3 - 4x \right]_{x=-1}^{x=2} dy \\ &= \int_{-2}^1 \left[\left(\frac{8}{3} - 2y + 2y^3 - 8 \right) - \left(-\frac{1}{3} - \frac{1}{2}y - y^3 + 4 \right) \right] dy \\ &= \int_{-2}^1 \left(-9 - \frac{3}{2}y + 3y^3 \right) dy \\ &= \left[-9y - \frac{3}{4}y^2 + \frac{3}{4}y^4 \right]_{-2}^1 \\ &= \left(-9 - \frac{3}{4} + \frac{3}{4} \right) - (18 - 3 + 12) \\ &= -36.\end{aligned}$$

Alternatively, we could begin by integrating with respect to y :

$$\begin{aligned}\iint_R (x^2 - xy + y^3 - 4) dA &= \int_{-1}^2 \int_{-2}^1 (x^2 - xy + y^3 - 4) dy dx \\ &= \int_{-1}^2 \left[x^2 y - \frac{1}{2} x y^2 + \frac{1}{4} y^4 - 4y \right]_{y=-2}^{y=1} dx \\ &= \int_{-1}^2 \left[\left(x^2 - \frac{1}{2} x + \frac{1}{4} - 4 \right) - (-2x^2 - 2x + 4 + 8) \right] dx \\ &= \int_{-1}^2 \left(3x^2 + \frac{3}{2} x - \frac{63}{4} \right) dx \\ &= \left[x^3 + \frac{3}{4} x^2 - \frac{63}{4} x \right]_{-1}^2 \\ &= \left(8 + 3 - \frac{63}{2} \right) - \left(-1 + \frac{3}{4} + \frac{63}{4} \right) \\ &= -36.\end{aligned}$$

4. If we start by integrating with respect to x , we have

$$\begin{aligned}V &= \iint_R \frac{x^2}{\sqrt{9-y^2}} dA \\ &= \int_0^{\frac{3}{2}} \int_{-3}^3 \frac{x^2}{\sqrt{9-y^2}} dx dy \\ &= \int_0^{\frac{3}{2}} \left[\frac{1}{3} \cdot \frac{x^3}{\sqrt{9-y^2}} \right]_{x=-3}^{x=3} dy \\ &= \int_0^{\frac{3}{2}} \left(\frac{9}{\sqrt{9-y^2}} + \frac{9}{\sqrt{9-y^2}} \right) dy \\ &= 18 \int_0^{\frac{3}{2}} \frac{1}{\sqrt{9-y^2}} dy \\ &= 18 \left[\arcsin \left(\frac{y}{3} \right) \right]_0^{\frac{3}{2}} \\ &= 18 \left[\arcsin \left(\frac{1}{2} \right) - \arcsin(0) \right] \\ &= 18 \cdot \frac{\pi}{6} \\ &= 3\pi.\end{aligned}$$