MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.4 | **Math 2000 Worksheet** | FALL 2018

SOLUTIONS

1. Observe that

$$\frac{\partial z}{\partial x} = \ln(x+2y) + \frac{x}{x+2y} \qquad \frac{\partial z}{\partial y} = \frac{2x}{x+2y}$$
$$\frac{dx}{dt} = \cos(t) \qquad \frac{dy}{dt} = -\sin(t)$$

so then

$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}\\ &= \left[\ln(x+2y) + \frac{x}{x+2y}\right]\cos(t) + \left[\frac{2x}{x+2y}\right][-\sin(t)]\\ &= \ln(x+2y)\cos(t) + \frac{x\cos(t)}{x+2y} - \frac{2x\sin(t)}{x+2y}\\ &= \ln(x+2y)\cos(t) + \frac{x\cos(t)-2x\sin(t)}{x+2y}.\end{aligned}$$

2. We have

$$\frac{\partial z}{\partial u} = \cos(u)\tan(v) \qquad \frac{\partial z}{\partial v} = \sin(u)\sec^2(v) \qquad \frac{\partial u}{\partial x} = 3$$
$$\frac{\partial v}{\partial x} = 1 \qquad \frac{\partial u}{\partial y} = 1 \qquad \frac{\partial v}{\partial y} = -1$$

and so

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x}\\ &= \cos(u)\tan(v)(3) + \sin(u)\sec^2(v)(1)\\ &= 3\cos(u)\tan(v) + \sin(u)\sec^2(v)\\ \frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y}\\ &= \cos(u)\tan(v)(1) + \sin(u)\sec^2(v)(-1)\\ &= \cos(u)\tan(v) - \sin(u)\sec^2(v).\end{aligned}$$

3. We have

$$\frac{\partial w}{\partial x} = \frac{z}{\sqrt{1-y^2}} \qquad \frac{\partial w}{\partial y} = \frac{xyz}{(1-y^2)^{\frac{3}{2}}} \qquad \frac{\partial w}{\partial z} = \frac{x}{\sqrt{1-y^2}}$$
$$\frac{\partial x}{\partial r} = 2r \qquad \frac{\partial z}{\partial r} = 4\theta e^{4r\theta} \qquad \frac{\partial y}{\partial \theta} = -\sin(\theta)$$
$$\frac{\partial z}{\partial \theta} = 4re^{4r\theta} \qquad \frac{\partial y}{\partial r} = \frac{\partial x}{\partial \theta} = 0$$

so that

$$\begin{aligned}
\frac{\partial w}{\partial r} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r} \\
&= \frac{z}{\sqrt{1-y^2}}(2r) + \frac{xyz}{(1-y^2)^{\frac{3}{2}}}(0) + \frac{x}{\sqrt{1-y^2}}(4\theta e^{4r\theta}) \\
&= \frac{2rz}{\sqrt{1-y^2}} + \frac{4x\theta e^{4r\theta}}{\sqrt{1-y^2}} \\
&= \frac{2rz + 4x\theta e^{4r\theta}}{\sqrt{1-y^2}} \\
\frac{\partial w}{\partial \theta} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \theta} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial \theta} \\
&= \frac{z}{\sqrt{1-y^2}}(0) + \frac{xyz}{(1-y^2)^{\frac{3}{2}}}[-\sin(\theta)] + \frac{x}{\sqrt{1-y^2}}(4re^{4r\theta}) \\
&= -\frac{xyz\sin(\theta)}{(1-y^2)^{\frac{3}{2}}} + \frac{4xre^{4r\theta}}{\sqrt{1-y^2}} \\
&= \frac{4x(1-y^2)re^{4r\theta} - xyz\sin(\theta)}{(1-y^2)^{\frac{3}{2}}}.
\end{aligned}$$

4. We write

$$F(x,y) = \sin(x) + \cos(y) - \sin(x)\cos(y) - 7$$

so that

$$F_x(x,y) = \cos(x) - \cos(x)\cos(y) \quad \text{and} \quad F_y(x,y) = -\sin(y) + \sin(x)\sin(y),$$

and hence

$$\frac{dy}{dx} = -\frac{F_x(x,y)}{F_y(x,y)} = -\frac{\cos(x) - \cos(x)\cos(y)}{-\sin(y) + \sin(x)\sin(y)} = \frac{\cos(x) - \cos(x)\cos(y)}{\sin(y) - \sin(x)\sin(y)}.$$

5. We set

$$F(x,y,z) = x^2 - \sqrt{y} + z^2 - 2xyz$$

so that

$$F_x(x,y,z) = 2x - 2yz \quad \text{and} \quad F_y(x,y,z) = -\frac{1}{2\sqrt{y}} - 2xz \quad \text{and} \quad F_z(x,y,z) = 2z - 2xy.$$

Thus

$$\begin{aligned}
\frac{\partial z}{\partial x} &= -\frac{F_x(x,y,z)}{F_z(x,y,z)} = -\frac{2x - 2yz}{2z - 2xy} = \frac{yz - x}{z - xy} \\
\frac{\partial z}{\partial y} &= -\frac{F_y(x,y,z)}{F_z(x,y,z)} = -\frac{-\frac{1}{2\sqrt{y}} - 2xz}{2z - 2xy} = \frac{1 + 4xz\sqrt{y}}{4\sqrt{y}(z - xy)}.
\end{aligned}$$