

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 1.5

Math 2000 Worksheet

FALL 2018

SOLUTIONS

1. (a) We use the Direct Comparison Test with the convergent p -series $\sum_{i=1}^{\infty} \frac{5}{i^3}$ (recalling that multiplying by the constant 5 doesn't affect the convergence of the series). Then for $i \geq 1$

$$i^3 < i^3 + 2$$

$$\frac{1}{i^3} > \frac{1}{i^3 + 2}$$

$$\frac{5}{i^3} > \frac{5}{i^3 + 2},$$

so the given series converges as well.

- (b) We use the Direct Comparison Test with the harmonic series $\sum_{i=2}^{\infty} \frac{1}{i}$, which is divergent. We have for $i \geq 2$,

$$i > i - \sqrt{i}$$

$$\frac{1}{i} < \frac{1}{i - \sqrt{i}}$$

and so the given series also diverges.

- (c) We use the Limit Comparison Test with the convergent p -series $\sum_{i=1}^{\infty} \frac{1}{i^2}$:

$$\lim_{i \rightarrow \infty} \frac{\frac{1}{3i^2 - 1}}{\frac{1}{i^2}} = \lim_{i \rightarrow \infty} \frac{i^2}{3i^2 - 1} = \frac{1}{3},$$

so the given series converges as well.

- (d) We use the Direct Comparison Test with the divergent geometric series $\sum_{i=1}^{\infty} \left(\frac{5}{4}\right)^i$. For $i \geq 1$,

$$5^i < 6 + 5^i$$

$$\frac{5^i}{4^i} < \frac{6 + 5^i}{4^i}$$

$$\left(\frac{5}{4}\right)^i < \frac{6 + 5^i}{4^i}.$$

Hence the given series is divergent.

- (e) We use the Limit Comparison Test with the convergent geometric series $\sum_{i=0}^{\infty} \left(\frac{5}{6}\right)^i = \sum_{i=0}^{\infty} \frac{5^i}{6^i}$:

$$\lim_{i \rightarrow \infty} \frac{\frac{1+5^i}{1+6^i}}{\frac{5^i}{6^i}} = \frac{\left(\frac{1}{5}\right)^i + 1}{\left(\frac{1}{6}\right)^i + 1} = 1,$$

so the given series is convergent.

- (f) We use the Limit Comparison Test with the (divergent) harmonic series $\sum_{i=1}^{\infty} \frac{1}{i}$:

$$\lim_{i \rightarrow \infty} \frac{\frac{i+4}{(i+2)^2}}{\frac{1}{i}} = \lim_{i \rightarrow \infty} \frac{i^2 + 4i}{i^2 + 4i + 4} = 1,$$

so the given series diverges.

- (g) We use the Direct Comparison Test with the convergent p -series $\sum_{i=1}^{\infty} \frac{1}{i^{\frac{5}{2}}} = \sum_{i=1}^{\infty} \frac{1}{\sqrt{i^5}}$. Then for $i \geq 1$,

$$\begin{aligned} i^5 + 1 &> i^5 \\ \sqrt{i^5 + 1} &> \sqrt{i^5} \\ \frac{1}{\sqrt{i^5 + 1}} &< \frac{1}{\sqrt{i^5}} \end{aligned}$$

so the given series is convergent.

- (h) We use the Limit Comparison Test with the divergent geometric series $\sum_{i=1}^{\infty} 2^i$:

$$\lim_{i \rightarrow \infty} \frac{\frac{2^i(4i^3-1)}{i^3+i^2+11}}{2^i} = \lim_{i \rightarrow \infty} \frac{4i^3-1}{i^3+i^2+11} = 4,$$

so the given series diverges too.

- (i) First note that

$$\frac{i!}{i^i} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdots i}{i \cdot i \cdot i \cdot i \cdots i} = \frac{1}{i} \cdot \frac{2}{i} \cdot \frac{3}{i} \cdot \frac{4}{i} \cdots \frac{k}{i} \cdots \frac{i}{i},$$

so every factor $\frac{k}{i} \leq 1$. Hence we can conclude that

$$\frac{i!}{i^i} \leq \frac{1}{i} \cdot \frac{2}{i} \cdot 1 \cdot 1 \cdots 1 = \frac{2}{i^2}.$$

Since $\sum_{i=1}^{\infty} \frac{2}{i^2}$ is a convergent p -series, by the Direct Comparison Test, the given series converges as well.