MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS



## SOLUTIONS

1. (a) We can write

$$\frac{(2i)!}{2 \cdot 4 \cdot 6 \cdots 2i} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdots (2i-1)(2i)}{2 \cdot 4 \cdot 6 \cdots 2i} = 1 \cdot 3 \cdot 5 \cdots (2i-1).$$

(b) Here,

$$\frac{2 \cdot 4 \cdot 6 \cdots (2i)}{5 \cdot 10 \cdot 15 \cdots (5i)} = \frac{2^i(1 \cdot 2 \cdot 3 \cdots i)}{5^i(1 \cdot 2 \cdot 3 \cdots i)} = \frac{2^i i!}{5^i i!} = \left(\frac{2}{5}\right)^i.$$

2. (a) $\{a_i\} = \left\{1, 0, -\frac{1}{9}, 0, \frac{1}{25}, \ldots\right\}$

(b) $\{a_i\} = \left\{2, -\frac{3}{2}, \frac{2}{3}, -\frac{5}{24}, \frac{1}{20}, \ldots\right\}$

(c) $\{a_i\} = \left\{4, \frac{2}{3}, \frac{1}{4}, \frac{1}{9}, \frac{1}{19}, \ldots\right\}$

3. (a) The numerator of each term is simply $i$, while the denominator in each case is a perfect cube, starting with $2^3$. Thus

$$a_i = \frac{i}{(i+1)^3}.$$

(b) This is an alternating sequence, so let's first consider the sequence consisting of the absolute value of each term,

$$\{p_i\} = \{3, 8, 13, 18, \ldots\}.$$

The difference between each term is 5, so

$$p_i = 5i - 2.$$

Thus, for the original sequence,

$$a_i = (-1)^i(5i - 2).$$

(c) It's obvious that the sequence is formed by alternately subtracting or adding 8 to the preceding term. However, this seems difficult to represent as an equation. It's easier to view this as an alternating sequence in which 4 is either added to or subtracted from 6. Thus

$$a_i = 6 + 4(-1)^{i+1}.$$