MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 1.11 | **Math 2000 Worksheet** | FALL 2018

SOLUTIONS

1. (a) $z + w = (3 - 4i) + (-1 + 7i) = (3 - 1) + (-4 + 7)i = 2 + 3i$

   (b) $z - w = (3 - 4i) - (-1 + 7i) = (3 + 1) + (-4 - 7)i = 4 - 11i$

   (c) $z \cdot w = (3 - 4i) \cdot (-1 + 7i) = -3 + 4i + 21i - 28i^2 = (-3 + 28) + (4 + 21)i = 25 + 25i$

   (d) $\dfrac{w}{z} = \dfrac{-1 + 7i}{3 - 4i} \cdot \dfrac{3 + 4i}{3 + 4i} = \dfrac{-3 + 21i - 4i + 28i^2}{9 - 16i^2} = \dfrac{(-3 - 28) + (21 - 4)i}{9 + 16} = \dfrac{31}{25} - \dfrac{17}{25}i$

   (e) $w^2 = (-1 + 7i)^2 = 1 - 7i - 7i + 49i^2 = (1 - 49) + (-7 - 7)i = -48 - 14i$

   (f) $|z| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$

2. We are given that $r = 4$ and $\theta = \frac{5\pi}{6}$. The real part of z is

$$\alpha = r\cos(\theta) = 4\cos\left(\frac{5\pi}{6}\right) = 4 \cdot \frac{-\sqrt{3}}{2} = -2\sqrt{3}.$$

   The imaginary part of z is

$$\beta = r\sin(\theta) = 4\sin\left(\frac{5\pi}{6}\right) = 4 \cdot \frac{1}{2} = 2.$$

   Thus

$$z = -2\sqrt{3} + 2i.$$

3. (a) The real part of z is $\alpha = \sqrt{3}$ while the imaginary part is $\beta = 1$. Thus the modulus is

$$r = \sqrt{\alpha^2 + \beta^2} = \sqrt{3 + 1} = 2$$

   while the argument is

$$\theta = \arctan\left(\frac{\beta}{\alpha}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}.$$

   Therefore the polar representation of z is $\left(2, \frac{\pi}{6}\right)$.

   (b) From part (a), we know that we can write $z = 2e^{\frac{\pi}{6}i}$ so

$$z^4 = \left(2e^{\frac{\pi}{6}i}\right)^4 = 16e^{\frac{2\pi}{3}i} = 16\left[\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right] = -8 + 8\sqrt{3}i.$$