

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 3

MATH 2000

FALL 2018

Due: Friday, October 5th, 2018 by 2:00pm. SHOW ALL WORK.

Note: You should complete the worksheets for Sections 1.2 and 2.3 before you work on this assignment.

1. Use any appropriate method to evaluate the limit, if it exists, of each of the following sequences $\{a_i\}$. If a sequence is divergent, explain why.

(a) $a_i = \frac{4 \cdot 8 \cdot 12 \cdots (4i)}{4^i}$

(b) $a_i = (-1)^{i+1} \frac{9 - 4^{i+2}}{4^{2i-1} + 3^i}$

(c) $a_i = (-1)^i \frac{7^i - 3^i}{7^i + 3^i}$

(d) $a_i = \frac{i^2 \cos(3i)}{4i^3 - 2i^2 + 5}$

(e) $a_i = (e^i + i)^{\frac{1}{i}}$

2. Show that each of the following sequences $\{a_i\}$ is monotonic (or has a monotonic tail) and is bounded, and therefore converges by the Bounded Monotonic Sequence Theorem.

(a) $a_i = \sqrt{i^2 + 8} - i$

(b) $a_i = \frac{(i!)^2}{(2i)!}$

3. Find the first- and second-order partial derivatives for the following functions.

(a) $f(x, y) = 2x^3y^2 - 7xy^3 + 9xy - 3x^2 - y + 5$

(b) $f(r, \theta) = \arctan(r\theta^2)$

4. Determine whether $z = 3xe^y - 7ye^x$ is a solution of the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - y \frac{\partial^3 z}{\partial x^2 \partial y} = x \frac{\partial^3 z}{\partial x \partial y^2} - \frac{\partial^3 z}{\partial y^3}.$$