8. a) Along $y=0$, the limit becomes

$$\lim_{x\to 0} \frac{0}{2x^4+0} = \lim_{x\to 0} 0 = 0$$

Along $x=0$, the limit becomes

$$\lim_{y\to 0} \frac{0}{0+3y^2} = \lim_{y\to 0} 0 = 0$$

Along $y=x$, the limit becomes

$$\lim_{x\to 0} \frac{5x^3}{2x^4+3x^2} = \lim_{x\to 0} \frac{5x}{2x^2+3} = \frac{0}{3} = 0$$

Along $y=x^2$, the limit becomes

$$\lim_{x\to 0} \frac{5x^4}{2x^4+3x^4} = \lim_{x\to 0} \frac{5x^4}{5x^4} = \lim_{x\to 0} 1 = 1$$

Because the limits along two paths are not equal, the limit of the function does not exist.

b) $$\frac{\partial z}{\partial t} = \cos(x-kt)\cdot(-k) - \sin(x+kt)\cdot k$$
$$= -k\cos(x-kt) - k\sin(x+kt)$$

$$\frac{\partial z}{\partial x} = \cos(x-kt)\cdot 1 - \sin(x+kt)\cdot 1$$
$$= \cos(x-kt) - \sin(x+kt)$$

$$\frac{\partial^2 z}{\partial t^2} = k\sin(x-kt)\cdot(-k) - k\cos(x+kt)\cdot k$$
$$= -k^2\sin(x-kt) - k^2\cos(x+kt)$$

$$\frac{\partial^2 z}{\partial x^2} = -\sin(x-kt)\cdot 1 - \cos(x+kt)\cdot 1$$
$$= -\sin(x-kt) - \cos(x+kt)$$

Then $$k^2\frac{\partial^2 z}{\partial x^2} = k^2\left[-\sin(x-kt) - \cos(x+kt)\right]$$
$$= -k^2\sin(x-kt) - k^2\cos(x+kt)$$
$$= \frac{\partial^2 z}{\partial t^2}$$ and so z does satisfy the PDE