4. a) We use the Root Test:

$$L = \lim_{i\to\infty} |a_i|^{1/i} = \lim_{i\to\infty} \frac{e^3 \cdot \left(\frac{i+5}{i}\right)}{i^{2/i}}$$

$$= e^3 \cdot \lim_{i\to\infty} \frac{1}{(i^{1/i})^2} = e^3 \cdot 1 = e^3 > 1$$

Thus the given series <u>diverges</u>

b) Observe that

$$0 \le \cos^2(i^3+1) \le 1$$

$$0 \le \frac{\cos^2(i^3+1)}{3^i} \le \frac{1}{3^i} = \left(\frac{1}{3}\right)^i$$

By the Direct Comparison Test with test series $\sum t_i = \sum \left(\frac{1}{3}\right)^i$ (a convergent geometric series), since $a_i \le t_i$ the given series must <u>converge</u>.

c) We use the Ratio Test with

$$|a_i| = \frac{1 \cdot 4 \cdot 7 \cdots (3i+1)}{5^i\, i!} \quad \text{and} \quad |a_{i+1}| = \frac{1 \cdot 4 \cdot 7 \cdots (3i+1)(3i+4)}{5^{i+1} \cdot (i+1)!}$$

So $$L = \lim_{i\to\infty} \left|\frac{a_{i+1}}{a_i}\right| = \lim_{i\to\infty} \frac{1 \cdot 4 \cdot 7 \cdots (3i+1)(3i+4)}{5^{i+1} \cdot (i+1)!} \cdot \frac{5^i\, i!}{1 \cdot 4 \cdot 7 \cdots (3i+1)}$$

$$= \lim_{i\to\infty} \frac{3i+4}{5(i+1)}$$

$$= \frac{3}{5} < 1$$

Thus the given series <u>converges</u>.