3. Let $f(x)=\frac{\ln(x)}{x^2}$ which is positive and continuous for $x \geq 1$.

Also, $f'(x)=\frac{\frac{1}{x}\cdot x^2 - 2x\ln(x)}{x^4} = \frac{1-2\ln(x)}{x^3} < 0$ for x sufficiently large so $f(x)$ is decreasing.

We must evaluate

$$\int_1^\infty \frac{\ln(x)}{x^2}\,dx = \lim_{T\to\infty} \int_1^T \frac{\ln(x)}{x^2}\,dx$$

We use integration by parts with

$w=\ln(x) \qquad dv=\frac{1}{x^2}dx$

$dw=\frac{1}{x}dx \qquad v=-\frac{1}{x}$

So the integral becomes

$$\lim_{T\to\infty}\left[\left[-\frac{1}{x}\cdot\ln(x)\right]_1^T + \int_1^T \frac{1}{x^2}\,dx\right]$$

$$= \lim_{T\to\infty}\left[-\frac{\ln(x)}{x} - \frac{1}{x}\right]_1^T$$

$$= \lim_{T\to\infty}\left[-\frac{\ln(T)}{T} - \frac{1}{T} + 1\right]$$

$$\overset{H}{=} 1 + \lim_{T\to\infty}\frac{-1/T}{1}$$

$$= 1 - \lim_{T\to\infty}\frac{1}{T}$$

$$= 1-0$$

$$= 1$$

By the Integral Test, $\sum_{i=1}^{\infty}\frac{\ln(i)}{i^2}$ must be convergent.