2. a) We rewrite the series as

$$\sum_{i=0}^{\infty} \frac{3^i \cdot 3^{-2}}{16^i} = \frac{1}{9} \sum_{i=0}^{\infty} \left(\frac{3}{16}\right)^i$$

$$= \frac{1}{9} \cdot \frac{1}{1-3/16}$$

$$= \frac{1}{9} \cdot \frac{16}{13} \boxed{= \frac{16}{117}}$$

b) We have

$$\frac{1}{i^2+15i+56} = \frac{1}{(i+8)(i+7)} = \frac{A}{i+8} + \frac{B}{i+7}$$

$$1 = A(i+7) + B(i+8)$$

For $i=-8$, we have $1 = A(-1) \rightarrow A = -1$

$i=-7$, we have $1 = B \cdot 1 \rightarrow B = 1$

So $\frac{1}{i^2+15i+56} = \frac{1}{i+7} - \frac{1}{i+8}$

Therefore $S_1 = \frac{1}{8} - \frac{1}{9}$

$$S_2 = \left(\frac{1}{8} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{10}\right) = \frac{1}{8} - \frac{1}{10}$$

$$S_3 = \left(\frac{1}{8} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{10}\right) + \left(\frac{1}{10} - \frac{1}{11}\right) = \frac{1}{8} - \frac{1}{11}$$

$\vdots$

$$S_n = \left(\frac{1}{8} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{10}\right) + \cdots + \left(\frac{1}{n+7} - \frac{1}{n+8}\right) = \frac{1}{8} - \frac{1}{n+8}$$

$$\lim_{n\to\infty} S_n = \frac{1}{8} - \lim_{n\to\infty} \frac{1}{n+8}$$

$$= \frac{1}{8} - 0$$

$$\boxed{= \frac{1}{8}}$$