Please remember that each final exam is different, and the style and emphasis of the questions will vary from semester to semester.

[4] 1. Find the limit of the sequence $\{a_i\} = \left\{ \left(1 - \frac{5}{i}\right)^i \right\}$ or explain why it does not exist.

2. Find the sum of each of the following convergent series.

[3] (a) $\displaystyle\sum_{i=0}^{\infty} \frac{3^{i-2}}{4^{2i}}$

[4] (b) $\displaystyle\sum_{i=1}^{\infty} \frac{1}{i^2 + 15i + 56}$

[5] 3. Use the Integral Test to determine the convergence or divergence of $\displaystyle\sum_{i=1}^{\infty} \frac{\ln(i)}{i^2}$. Remember to verify the conditions of the Integral Test.

4. Use an appropriate test to determine the convergence or divergence of the following series. Identify the tests used.

[4] (a) $\displaystyle\sum_{i=1}^{\infty} \frac{e^{3i}\left(\frac{i+5}{i}\right)^i}{i^2}$

[4] (b) $\displaystyle\sum_{i=1}^{\infty} \frac{\cos^2(i^3 + 1)}{3^i}$

[4] (c) $\displaystyle\sum_{i=0}^{\infty} (-1)^i \frac{1 \cdot 4 \cdot 7 \cdots (3i + 1)}{5^i i!}$

[7] 5. Determine whether the series $\displaystyle\sum_{i=1}^{\infty} \frac{(-1)^i}{4i + 3}$ converges absolutely, converges conditionally, or diverges. Justify your answer.

[7] 6. Find the interval of convergence of the power series $\displaystyle\sum_{i=1}^{\infty} \frac{i}{5^i(i^2 + 2)}(x + 3)^i$.

[4] 7. Use the definition to find the first four non-zero terms of the Taylor polynomial, centered at $x = 1$, for $f(x) = e^{2x}$.

[4] 8. (a) Evaluate $\lim_{(x,y)\to(0,0)} \frac{5x^2y}{2x^4+3y^2}$ or show that the limit does not exist.

[6] (b) Show that the function $z = \sin(x - kt) + \cos(x + kt)$, where k is a constant, satisfies the equation

$$\frac{\partial^2 z}{\partial t^2} = k^2 \frac{\partial^2 z}{\partial x^2}.$$

[6] 9. (a) Given $w = x^2 - y^2 + z^2$ where $x = t^2 + s^2$, $y = t^2 - s^2$ and $z = st$, use the Chain Rule to find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$.

[5] (b) The equation $xe^y + y^2 \ln(x) + z^2 y = 8z$ defines z implicitly as a function of x and y. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

[7] 10. Find any critical points and then use the Second Derivatives Test to determine any local extrema or saddle points of the function $f(x, y) = \frac{x^3}{3} + y^2 + 2xy - 6x - 3y + 5$.

11. Use a double integral to find the area of D in each case.

[5] (a) D is the region bounded by the curves $x - y^2 - 1 = 0$ and $x - 4y - 6 = 0$

[7] (b) D is the circle $x^2 + y^2 - 6x = 0$

[7] 12. Evaluate $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{1 + x^3}\, dx\, dy$ by reversing the order of integration. Sketch the region of integration.

[7] 13. Use polar coordinates to evaluate $\iint_R e^{-3(x^2+y^2)}\, dA$ where R is the region in the first quadrant bounded by the lines $y = 0$, $x = 0$, and the circle $x^2 + y^2 = 3$.