

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.2

Math 1090 Worksheet

FALL 2026

SOLUTIONS

1. (a) Any value of x will be in D as long as it does not make the denominator zero. In other words, we must solve the equation

$$\begin{aligned}x^2 + 4x - 12 &= 0 \\(x + 6)(x - 2) &= 0\end{aligned}$$

so $x = -6$ and $x = 2$. Hence

$$D = \{x \mid x \neq -6, x \neq 2\}.$$

- (b) The square root is undefined if its argument is negative, so we require $7 - x \geq 0$ and thus $x \leq 7$. But if $x = 7$ then the denominator of f will be zero, so we must exclude this value from the domain as well. Hence

$$D = (-\infty, 7).$$

- (c) We must have $9x^2 - 4 \geq 0$. We solve the corresponding equation

$$\begin{aligned}9x^2 - 4 &= 0 \\(3x - 2)(3x + 2) &= 0\end{aligned}$$

and see that the critical numbers are $x = \frac{2}{3}$ and $x = -\frac{2}{3}$ (both of which satisfy the inequality). For the interval $(-\infty, -\frac{2}{3})$ we substitute $x = -1$ into the original inequality and see that $9(-1)^2 - 4 = 5 \geq 0$, so the inequality is satisfied. For $(-\frac{2}{3}, \frac{2}{3})$ we try $x = 0$ which yields $0^2 - 4 = -4 < 0$, and hence the inequality is not satisfied. Finally, for $(\frac{2}{3}, \infty)$ we try $x = 1$ which gives $9(1)^2 - 4 = 5 \geq 0$, and again the inequality is satisfied. Hence the domain of the function is

$$D = \left(-\infty, -\frac{2}{3}\right] \cup \left[\frac{2}{3}, \infty\right).$$

- (d) Let's consider the square roots separately. The first one tells us that we need

$$x - 3 \geq 0,$$

so $x \geq 3$. The second one tells us that we need

$$3 - x \geq 0,$$

so $x \leq 3$. Both of these requirements have to hold, and the only value of x which satisfies the two restrictions is the single point $x = 3$. Thus the domain of the function is $D = \{3\}$. (This is called a **singleton set**, because it only includes one element.)

- (e) We must identify all values of x such that

$$|x - 3| - 1 = 0$$

$$|x - 3| = 1$$

so either $x - 3 = 1$ (and hence $x = 4$) or $x - 3 = -1$ (implying $x = 2$). Thus the domain of f is

$$D = \{x \mid x \neq 2, x \neq 4\}.$$

- (f) We must identify all values of x such that

$$|x - 3| + 1 = 0$$

$$|x - 3| = -1$$

but this can never happen because an absolute value is never negative. Hence f is defined for all x , and therefore $D = \mathbb{R}$.

2. (a) Since $-2 < -1$ we use the first part of the piecewise definition:

$$f(-2) = 3(-2)^2 = 12.$$

- (b) Since $-1 \leq -1 < 4$ we use the second part of the piecewise definition:

$$f(-1) = -1 + 5 = 4.$$

- (c) Since $-1 \leq 0 < 4$ we use the second part of the piecewise definition:

$$f(0) = 0 + 5 = 5.$$

- (d) None of the three parts of the piecewise definition apply when $x = 4$ so $f(4)$ is undefined.

- (e) Since $5 > 4$ we use the third part of the piecewise definition:

$$f(5) = 7.$$

3. (a) We have

$$f(-x) = (-x)^5 + (-x)^3 + (-x) = -x^5 - x^3 - x = -(x^5 + x^3 + x) = -f(x).$$

Thus $f(x)$ is odd.

- (b) We have

$$g(-x) = (-x)^4 + (-x)^3 + (-x) = x^4 - x^3 - x.$$

Clearly, $g(-x) \neq g(x)$, so $g(x)$ is not even. Furthermore, $g(-x) \neq -g(x)$, so $g(x)$ is not odd. Instead, $g(x)$ is neither even nor odd.

- (c) We have

$$h(-x) = (-x)^4 + (-x)^2 + 1 = x^4 + x^2 + 1 = h(x)$$

so $h(x)$ is even.

4. (a) Here

$$f(x+h) = 7 - 3(x+h) = 7 - 3x - 3h$$

so the difference quotient is

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{(7 - 3x - 3h) - (7 - 3x)}{h} \\ &= \frac{7 - 3x - 3h - 7 + 3x}{h} \\ &= \frac{-3h}{h} \\ &= -3.\end{aligned}$$

(b) Here

$$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$$

so the difference quotient is

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{(x^2 + 2xh + h^2) - x^2}{h} \\ &= \frac{2xh + h^2}{h} \\ &= \frac{h(2x + h)}{h} \\ &= 2x + h.\end{aligned}$$

(c) This time

$$f(x+h) = \frac{1}{(x+h)^2}$$

so the difference quotient is

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \\&= \frac{1}{h} \cdot \left(\frac{1}{(x+h)^2} \cdot \frac{x^2}{x^2} - \frac{1}{x^2} \cdot \frac{(x+h)^2}{(x+h)^2} \right) \\&= \frac{1}{h} \cdot \frac{x^2 - (x+h)^2}{x^2(x+h)^2} \\&= \frac{x^2 - (x^2 + 2xh + h^2)}{hx^2(x+h)^2} \\&= \frac{x^2 - x^2 - 2xh - h^2}{hx^2(x+h)^2} \\&= \frac{-2xh - h^2}{hx^2(x+h)^2} \\&= \frac{h(-2x - h)}{hx^2(x+h)^2} \\&= \frac{-2x - h}{x^2(x+h)^2}.\end{aligned}$$