

Section 2.4

Given a linear function $f(x) = mx + b$, the difference quotient over any interval is always m . Thus the average rate of change for a linear function is the same over any interval.

Def'n: The difference quotient of a linear function is its slope, so

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

where $P(x_1, y_1)$ and $Q(x_2, y_2)$ are any two points on the line.

eg Find the slope of the line that passes through the points $(5, -2)$ and $(3, 6)$.

$$m = \frac{6 - (-2)}{3 - 5} = \frac{8}{-2} \boxed{= -4}$$

If $m > 0$ then the line rises or increases from left to right.

If $m < 0$ then the line falls or decreases from left to right.

If $m=0$ then the line is horizontal.

For a vertical line, the slope m is undefined.

If we set $x=0$ in a linear function $f(x) = mx + b$, we obtain $f(0) = m \cdot 0 + b = b$ which means that the point $(0, b)$ is the y -intercept.

Thus the equation $y = mx + b$ is known as the slope-intercept of the line.

It is often convenient to first write the equation of a line in point-slope form: given a point $P(x_1, y_1)$ on a line with slope m , this is

$$y - y_1 = m(x - x_1)$$

Once we have obtained point-slope form, we can then rewrite the equation in slope-intercept form,

eg Find an equation of the line with slope $\frac{1}{2}$ through the point $(-4, 1)$.

In point-slope form, we have

$$y - 1 = \frac{1}{2}(x - (-4))$$

$$y - 1 = \frac{1}{2}x + 2$$

$$\boxed{y = \frac{1}{2}x + 3}$$

The general form of the equation of a line is

$$Ax + By + C = 0$$

for real numbers A, B, C . We typically try to make A, B, C integers.

eg The line $y = \frac{1}{2}x + 3$ can be written in the general form

$$\frac{1}{2}x - y + 3 = 0$$

$$2 \cdot (\frac{1}{2}x - y + 3) = 0 \cdot 2$$

$$\boxed{x - 2y + 6 = 0}$$

eg Find the slope and the y -intercept of the line $3x + 4y - 8 = 0$.

We convert to slope-intercept form by solving

for y :

$$4y = -3x + 8$$

$$\boxed{y = -\frac{3}{4}x + 2}$$

The slope is $m = -\frac{3}{4}$ and the y -intercept is $(0, 2)$.

Def'n : Two lines are parallel if they have the same slope, or if both are vertical.

eg Find an equation of the line l through the point $(1, -3)$ that is parallel to the line $4x + 2y + 5 = 0$.

The given line can be rewritten as

$$2y = -4x - 5$$

$$y = -2x - \frac{5}{2}$$

so its slope is -2 .

Thus the slope of l is $m = -2$ and so its

equation is

$$y - (-3) = -2(x - 1)$$

$$y + 3 = -2x + 2$$

$$\boxed{y = -2x - 1}$$

Def'n: Two lines are perpendicular if their slopes m_1 and m_2 are negative reciprocals, that is $m_2 = -\frac{1}{m_1}$. Two lines are also perpendicular if one is horizontal and the other is vertical.

eg Find an equation of the line l through the point $(1, -3)$ that is perpendicular to the line $4x + 2y + 5 = 0$.

We have already shown that the slope of the given line is -2 , so the slope of l is

$$m = -\frac{1}{-2} = \frac{1}{2}.$$

The equation of l is

$$y - (-3) = \frac{1}{2}(x - 1)$$

$$y + 3 = \frac{1}{2}x - \frac{1}{2}$$

$$\boxed{y = \frac{1}{2}x - \frac{7}{2}}$$

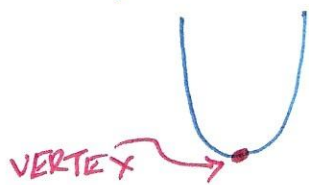
Section 2.5: Graphing Functions

Def'n: A quadratic function is any function that can be written in the form

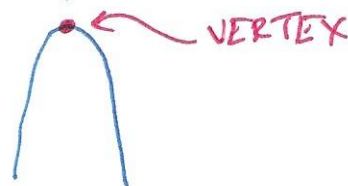
$$f(x) = ax^2 + bx + c$$

for $a \neq 0$.

The graph of a quadratic function is a parabola, which looks like



or



The equation of a parabola can be written as $y = ax^2 + bx + c$ or in standard form as $y = a(x-h)^2 + k$.

Here (h, k) are the coordinates of the vertex, with

$$h = -\frac{b}{2a}.$$

eg Find the vertex of the parabola with equation $y = x^2 + 2x - 3$.

Here $a = 1$, $b = 2$, $c = -3$ so

$$h = -\frac{b}{2a} = -\frac{2}{2 \cdot 1} = -\frac{2}{2} = -1$$

$$\text{and thus } k = f(h) = f(-1) = (-1)^2 + 2(-1) - 3 = -4$$

The vertex is the point $\boxed{(-1, -4)}$.