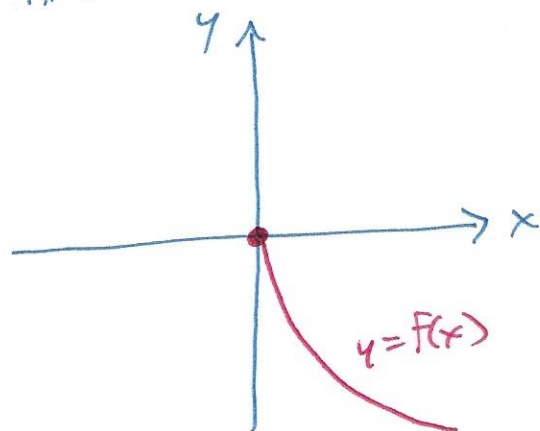


Section 2.3

Given a graph of a function, we can use it to determine various characteristics of that function.

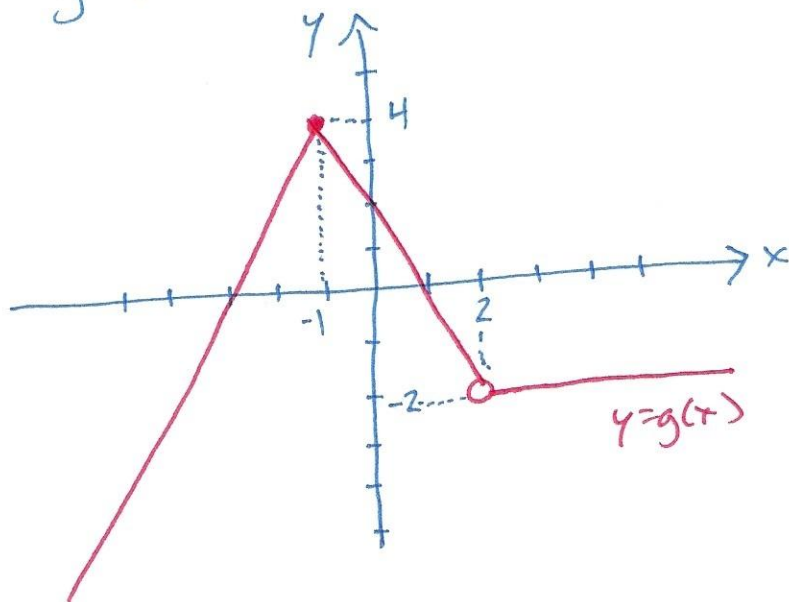
eg Find the domain and range of $y = f(x)$.



The domain is $[0, \infty)$.

The range is $(-\infty, 0]$.

eg Find the domain and range of $y = g(x)$.



The domain is $\{x \mid x \neq 2\}$.

The range is $(-\infty, 4]$.

Def'n: An x-intercept is a point on the graph which lies on the x-axis, so it has the form $(b, 0)$.

A y-intercept is a point on the graph which lies on the y-axis, so it has the form $(0, c)$.

eg For $y = g(x)$ from the last example, we can see that both $(-3, 0)$ and $(1, 0)$ are x-intercepts, while $(0, 2)$ is the y-intercept.

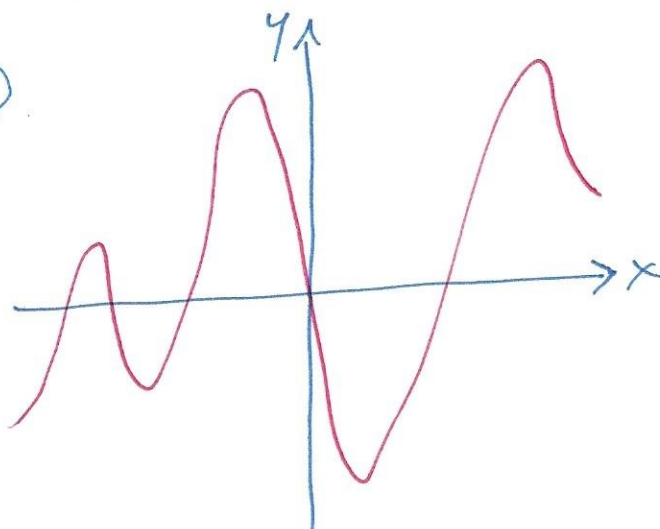
A function can have any number of x-intercepts, but can have no more than one y-intercept.

More generally, given an x-coordinate $x = b$ there can be at most one point on the graph of a function with that x-coordinate.

This leads to the Vertical Line Test: a graph represents a function if and only if any vertical line drawn through the graph intersects it no more than once.

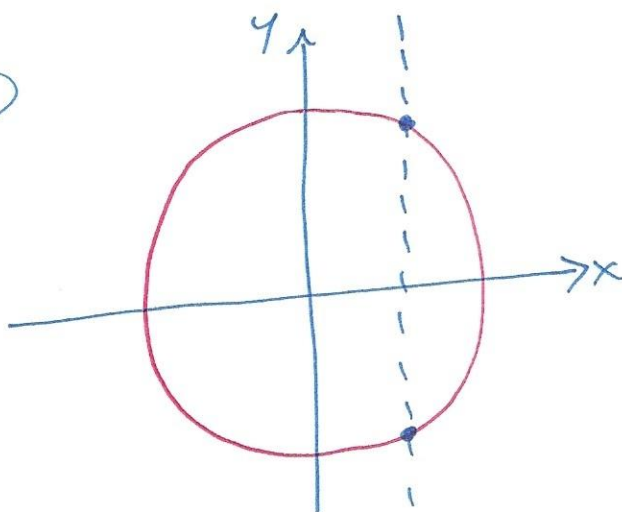
eg Determine which of the following is the graph of a function.

①



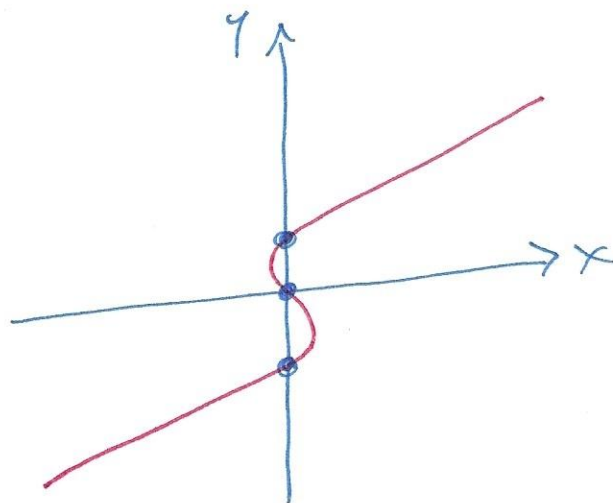
This passes the Vertical Line Test, so it is the graph of a function.

②



This fails the Vertical Line Test, so it is not the graph of a function.

③



This fails the Vertical Line Test, so it is not the graph of a function.

Section 2.4: Lines

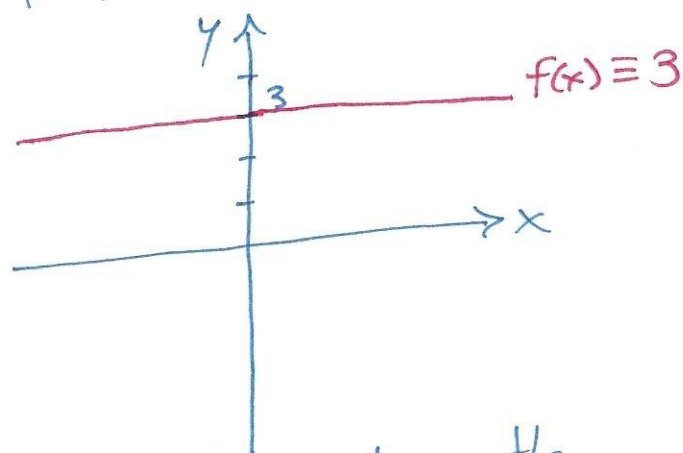
Def'n: A constant function is a function that can be written in the form $f(x) = k$ for a real number k . It is often denoted by $f(x) \equiv k$.

eg The constant function $f(x) \equiv 3$ is a function in which every point has a y -coordinate of 3.

The domain of a constant function is always $\mathbb{R}$.

The graph of a constant function must be a horizontal line.

eg $f(x) \equiv 3$



For a constant function, the point $(0, k)$ is always the y -intercept. A constant function has no x -intercepts unless $k=0$, in which case every point is an x -intercept. Thus the line $f(x) \equiv 0$ or $y=0$ is the x -axis.

Def'n: A linear function is a function that can be written in the form $f(x) = mx + b$, for real numbers m, b .

The graph of a linear function is always a diagonal line. It has exactly one x -intercept and one y -intercept.

eg Sketch the graph of $f(x) = \frac{1}{2}x - 1$.

To find the x -intercept, we set

$$\frac{1}{2}x - 1 = 0$$

$$\frac{1}{2}x = 1$$

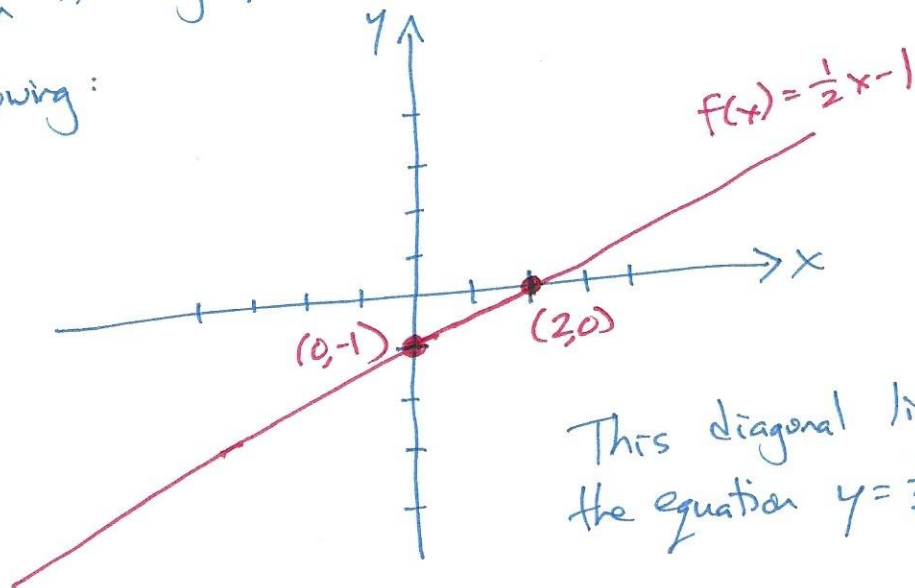
$$x = 2 \quad \text{so } (2, 0) \text{ is the } x\text{-intercept.}$$

To find the y -intercept, we evaluate

$$f(0) = \frac{1}{2} \cdot 0 - 1 = -1$$

so $(0, -1)$ is the y -intercept.

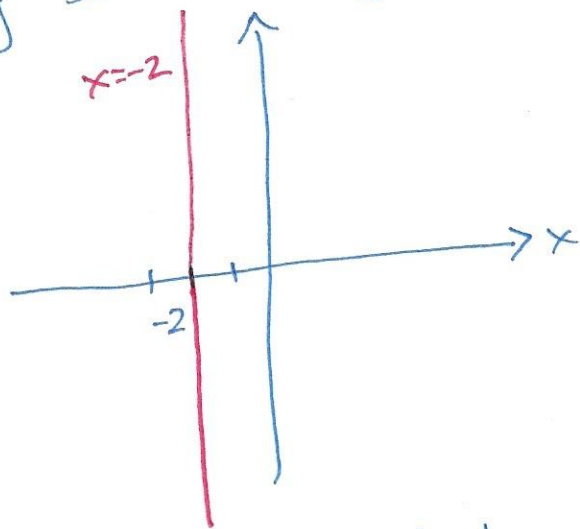
Then the graph of $f(x) = \frac{1}{2}x - 1$ is the following:



This diagonal line has the equation $y = \frac{1}{2}x - 1$.

A vertical line cannot be the graph of a function, because it fails the Vertical Line Test by definition. However, much like a horizontal line has an equation of the form $y=k$, a vertical line must have an equation of the form $x=k$.

eg Sketch the graph of the equation $x=-2$.



The y-axis is the vertical line with equation $x=0$.