

Section 2.2

Def'n: A function $f(x)$ is even if $f(-x) = f(x)$.

eg The function $f(x) = x^2 + 2$ is even because

$$f(-x) = (-x)^2 + 2 = x^2 + 2 = f(x).$$

eg The function $f(x) = x^2 + 2x - 3$ is not even because

$$\begin{aligned} f(-x) &= (-x)^2 + 2(-x) - 3 \\ &= x^2 - 2x - 3 \\ &\neq f(x). \end{aligned}$$

Def'n: A function $f(x)$ is odd if $f(-x) = -f(x)$.

eg The function $f(x) = \frac{5x}{x^2 + 9}$ is odd because

$$\begin{aligned} f(-x) &= \frac{5(-x)}{(-x)^2 + 9} = \frac{-5x}{x^2 + 9} \\ &= -\frac{5x}{x^2 + 9} = -f(x). \end{aligned}$$

Suppose we want to determine the average rate of change of a function $y = f(x)$ on an interval $[a, b]$.

Let Δx be the change in x over this interval, so $\Delta x = b - a$. Let Δy be the change in y over this interval, so $\Delta y = f(b) - f(a)$. Then the average rate of change is given by

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}.$$

We could instead assume that the interval starts at any point x and has length h , so the interval could instead be written $[x, x+h]$.

$$\text{Then } \Delta x = (x+h) - x = h$$

$$\Delta y = f(x+h) - f(x)$$

Now we have

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

and we call this the difference quotient.

eg Find the difference quotient for $f(x) = 4x + 3$.

$$\text{Here } f(x+h) = 4(x+h) + 3 = 4x + 4h + 3$$

Thus the difference quotient is

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{f(x+h) - f(x)}{h} = \frac{(4x + 4h + 3) - (4x + 3)}{h} \\ &= \frac{4x + 4h + 3 - 4x - 3}{h} \\ &= \frac{4h}{h}\end{aligned}$$

$$\boxed{= 4}$$

eg Find the difference quotient for $f(x) = \frac{x}{x-2}$.

$$\text{Here } f(x+h) = \frac{x+h}{(x+h)-2} = \frac{x+h}{x+h-2} \quad \text{so}$$

$$\frac{\Delta y}{\Delta x} = \frac{\frac{x+h}{x+h-2} - \frac{x}{x-2}}{h}$$

$$= \frac{1}{h} \cdot \left(\frac{x+h}{x+h-2} - \frac{x}{x-2} \right)$$

$$= \frac{1}{h} \cdot \left(\frac{x+h}{x+h-2} \cdot \frac{x-2}{x-2} - \frac{x}{x-2} \cdot \frac{x+h-2}{x+h-2} \right)$$

$$= \frac{1}{h} \cdot \left(\frac{x^2 + hx - 2x - 2h}{(x-2)(x+h-2)} - \frac{x^2 + hx - 2x}{(x-2)(x+h-2)} \right)$$

$$= \frac{1}{h} \cdot \left(\frac{x^2 + hx - 2x - 2h - x^2 - hx + 2x}{(x-2)(x+h-2)} \right)$$

$$= \frac{-2h}{h(x-2)(x+h-2)}$$

$$\boxed{= \frac{-2}{(x-2)(x+h-2)}}$$

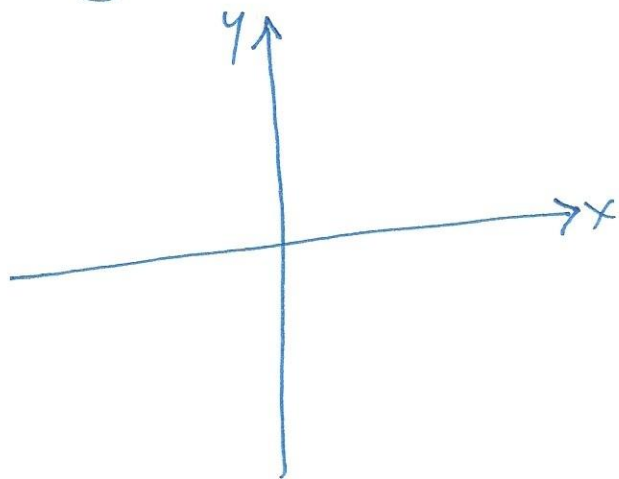
Observe that there will always be a common factor of h which can be extracted from $f(x+h)-f(x)$ and cancelled with the factor of h in the denominator of the difference quotient.

Section 2.3: Graphs of Functions

When we write a function in the form $y=f(x)$ we obtain a value of y in the range for each value of x in the domain. Thus we call x the independent variable, and we call y the dependent variable.

We can then think of the function as a collection of ordered pairs (x,y) .

Then we can visually represent the function by first drawing two number lines which meet at right angles.



The horizontal number line is called the x-axis.

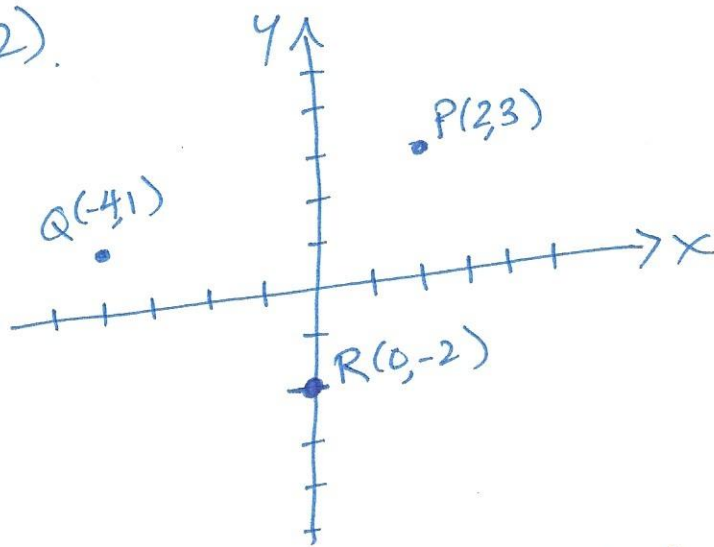
The vertical number line is called the y-axis.

This forms the Cartesian plane.

The point where the axes meet is taken to be the point where $x=0$ and where $y=0$. It is called the origin.

Now consider an ordered pair (b, c) generated by a function $y = f(x)$. It can be represented as a point on the Cartesian plane. We start at the origin, and move $|b|$ units to the right if $b > 0$ or to the left if $b < 0$. Then we move $|c|$ units up if $c > 0$ or down if $c < 0$.

eg Plot the points $P(2, 3)$, $Q(-4, 1)$ and $R(0, -2)$.



We say that the point (b, c) has x-coordinate b and y-coordinate c .

If we plot all of the ordered pairs corresponding to the domain of the function, we obtain its graph.