MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 4.2 **Math 1001 Worksheet** WINTER 2023

For practice only. Not to be submitted.

1. Find the particular solution to each initial value problem.

   (a) $t^2y^2\dfrac{dy}{dt} = 1, \quad y(3) = 1$

   (b) $t^2y^2\dfrac{dy}{dt} = \sqrt{1-y^2}, \quad y(-2) = 0$

   (c) $\dfrac{dy}{dt} - ty^2 - 4t = 0, \quad y(1) = 2$

   (d) $y\dfrac{dy}{dt} - e^{t+y} = 0, \quad y(0) = 0$

   (e) $\cos(y)\dfrac{dy}{dt} + \csc(y) = 0, \quad y\left(-\frac{1}{8}\right) = \frac{\pi}{6}$

2. Once we have solved a differential equation, we can apply the solution to solve problems involving any relevant scenario. Consider a population of parakeets which has been introduced to a tropical island, where they are growing at an exponential rate. Then we know the population can be modelled by the function

$$y(t) = y_0e^{kt}$$

   where $y(t)$ is the number of parakeets. Suppose t is measured in years, y_0 is the size of the initial population, and k is a constant of proportionality. Two years later, a group of "castaways" arrives on the island for a reality game show. During a challenge, they count roughly 50 parakeets. Three years later, some of the "castaways" return to the island for an "all-star" edition of the show. They discover that there are now about 150 parakeets.

   (a) Determine the value of y_0.

   (b) Determine the size of the parakeet population after another seven years have elapsed (that is, twelve years after the parakeets were introduced to the island).