MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.2 **Math 1001 Worksheet** WINTER 2023

For practice only. Not to be submitted.

1. Evaluate each of the following trigonometric integrals.

   (a) $\displaystyle\int_0^{\frac{\pi}{9}} \sin^2(6x)\cos^3(6x)\,dx$

   (b) $\displaystyle\int \sin^3(x)\cos^8(x)\,dx$

   (c) $\displaystyle\int \sin^2(x)\cos^5(x)\,dx$

   (d) $\displaystyle\int \frac{\cos^3(\ln(x))}{x}\,dx$

   (e) $\displaystyle\int x\sin^2(x)\,dx$

   (f) $\displaystyle\int \frac{1-\tan^2(x)}{\sec^2(x)}\,dx$

2. Strategies similar to those introduced for integrals of the form $\displaystyle\int \sin^m(x)\cos^n(x)\,dx$ can also work for combinations of $\sec(x)$ and $\tan(x)$ functions, and for combinations of $\csc(x)$ and $\cot(x)$.

   (a) Consider $\displaystyle\int \tan^5(x)\sec^5(x)\,dx$. Evaluate the integral as follows:

   - set aside a factor of $\sec(x)\tan(x)$
   - transform the remaining factors of $\tan(x)$ into $\sec(x)$ using the identity $\tan^2(x)+1 = \sec^2(x)$
   - use u-substitution with $u = \sec(x)$.

   (b) Consider $\displaystyle\int \frac{\cos^2(x)}{\sin^6(x)}\,dx$. Although this integral involves $\sin(x)$ and $\cos(x)$ functions, it cannot be evaluated using the techniques introduced in class. Show that it can be evaluated as follows:

   - rewrite the integrand in terms of $\cot(x)$ and $\csc(x)$ functions
   - set aside a factor of $\csc^2(x)$
   - transform the remaining factors of $\csc(x)$ into $\cot(x)$ using the identity $1+\cot^2(x) = \csc^2(x)$
   - use u-substitution with $u = \cot(x)$.