

MEMORIAL UNIVERSITY OF NEWFOUNDLAND  
DEPARTMENT OF MATHEMATICS AND STATISTICS

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SECTION 2.1

Math 1001 Worksheet

WINTER 2023

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**For practice only. Not to be submitted.**

1. Write each of the following sums using sigma notation (with  $i = 1$  as the lower bound of summation).

(a)  $\frac{2}{5} + \frac{4}{10} + \frac{8}{15} + \frac{16}{20} + \cdots + \frac{2^n}{5n}$

(b)  $y^2 + 8y^2 + 27y^2 + 64y^2 + \cdots + n^3y^2$

2. Use the summation formulas to evaluate each of the following sums.

(a)  $\sum_{i=1}^n (4i + 3)$

(b)  $\sum_{i=1}^n (i^3 - 6i)$

(c)  $\sum_{i=1}^n (3i + 1)^2$

3. Consider the region  $R$  which lies between the graph of  $f(x) = 2 - x$  and the  $x$ -axis, on the interval  $[-1, 1]$ .

- (a) Determine the width  $\Delta x$  of each subinterval in a regular partition of  $[-1, 1]$ .
- (b) Give an expression for the right endpoint  $x_i$  of the  $i$ th subinterval. Use this to find formulas for the point  $m_i$  and  $M_i$  at which  $f(x)$  attains its minimum and maximum values on the  $i$ th subinterval.
- (c) Determine  $f(m_i)$  and  $f(M_i)$ .
- (d) Find formulas for the lower sum  $s(n)$  and the upper sum  $S(n)$ .
- (e) Estimate the area  $A$  of the region  $R$  by finding the lower and upper sums with  $n = 5$  subintervals. Give your answers to one decimal place.
- (f) Estimate the area  $A$  of the region  $R$  by finding the lower and upper sums with  $n = 500$  subintervals. Give your answers to two decimal places.
- (g) Find the true value of  $A$  by taking the limit of the formulas you derived in part (d) as  $n \rightarrow \infty$ , and showing that these limits are equal.

4. Use the limit of a Riemann sum to determine the area  $A$  of the region which lies below the curve  $f(x) = 3 + 3x - x^2$ , above the  $x$ -axis, to the left of  $x = -1$  and to the right of  $x = 3$ .